# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 1**

**Question:**

Define the terms
**a** bipartite graph,
**b** alternating path,
**c** matching,
**d** complete matching. ***E***

**Solution:**

**a** A graph consisting of
- two distinct sets of vertices X and Y in which
- arcs can only join a vertex in X to a vertex in Y.

**b**
- A path from an unmatched vertex in X to an unmatched vertex in Y
- which alternately uses arcs not in/in the matching.

Two points to make in each of these responses.

**c** The one-to-one pairing of some elements of X with elements of Y.

**d** A one-to-one matching between all elements of X and Y.

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 2**

**Question:**

The organiser of a sponsored walk wishes to allocate each of six volunteers, Alan, Geoff, Laura, Nicola, Philip and Sam to one of the checkpoints along the route. Two volunteers are needed at checkpoint 1 (the start) and one volunteer at each of checkpoints 2, 3, 4 and 5 (the finish). Each volunteer will be assigned to just one checkpoint. The table shows the checkpoints each volunteer is prepared to supervise.

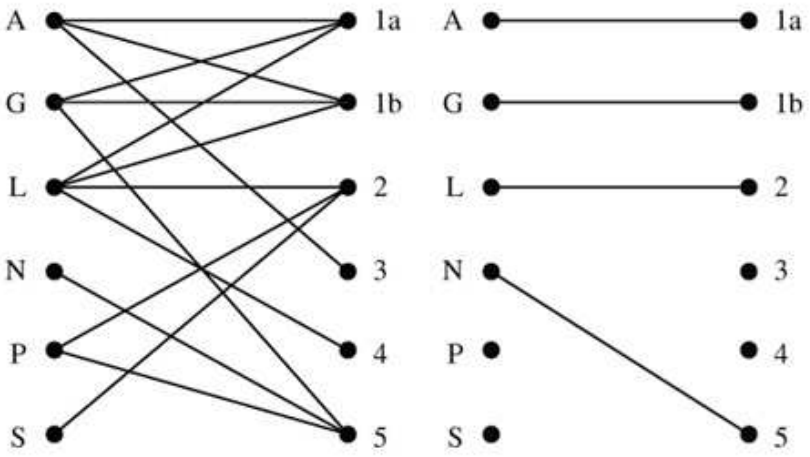
| Name | Checkpoints |
|---|---|
| Alan | 1 or 3 |
| Geoff | 1 or 5 |
| Laura | 2, 1 or 4 |
| Nicola | 5 |
| Philip | 2 or 5 |
| Sam | 2 |

Initially Alan, Geoff, Laura and Nicola are assigned to the first checkpoint in their individual list.

**a** Draw a bipartite graph to model this situation and indicate the initial matching in a distinctive way.
**b** Starting from this initial matching, use the maximum matching algorithm to find an improved matching. Clearly list any alternating paths you use.
**c** Explain why it is not possible to find a complete matching. ***E***

**Solution:**

**a**




**b** Six possible alternating paths – only one needed

**i** P – 2 = L – 4 change status P = 2 – L = 4
**ii** P – 2 = L – 1 = G – 4 change status P = 2 – L = 1 – G = 4
**iii** P – 2 = L – 1 = A – 3 change status P = 2 – L = 1 – A = 3
**iv** S – 2 = L – 4 change status S = 2 – L = 4
**v** S – 2 = L – 1 = G – 4 change status S = 2 – L = 1 – G = 4
**vi** S – 2 = L – 1 = A – 3 change status S = 2 – L = 1 – A = 3

Giving matchings as follows

Giving matchings as follows

| Person | (i) | (ii) | (iii) | (iv) | (v) | (vi) |
|---|---|---|---|---|---|---|
| A | 1 | 1 | 3 | 1 | 1 | 3 |
| G | 1 | 4 | 1 | 1 | 4 | 1 |
| L | 4 | 1 | 1 | 4 | 1 | 1 |
| N | 5 | 5 | 5 | 5 | 5 | 5 |
| P | 2 | 2 | 2 | | | |
| S | | | | 2 | 2 | 2 |

**c** For example, N must do 5 and S must do 2. This leaves P without a task.

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 3**

**Question:**

Ann, Bryn, Daljit, Gareth and Nickos have all joined a new committee. Each of them is to be allocated to one of five jobs 1, 2, 3, 4 or 5. The table shows each member's preferences for the jobs.

| | |
|---|---|
| **Ann** | 1 or 2 |
| **Bryn** | 3 or 1 |
| **Daljit** | 2 or 4 |
| **Gareth** | 5 or 3 |
| **Nickos** | 1 or 2 |

Initially Ann, Bryn, Daljit and Gareth are allocated the first job in their lists shown in the table.

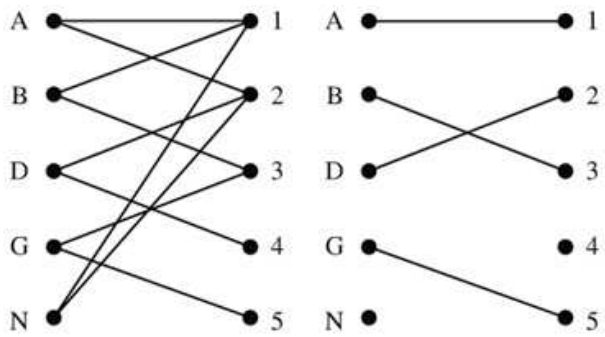
**a** Draw a bipartite graph to model the preferences shown in the table and indicate, in a distinctive way, the initial allocation of jobs.

**b** Use the matching improvement algorithm to find a complete matching, showing clearly your alternating path.

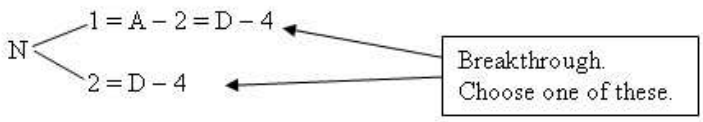
**c** Find a second alternating path from the initial allocation. ***E***

**Solution:**

**a**




**b** Possible paths




Change status
N = 1 – A = 2 – D = 4 giving matching A = 2 B = 3 D = 4 G = 5 N = 1
or
N = 2 – D = 4 giving matching A = 1 B = 3 D = 4 G = 5 N = 2

**c** Give the other alternating path.

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 4**

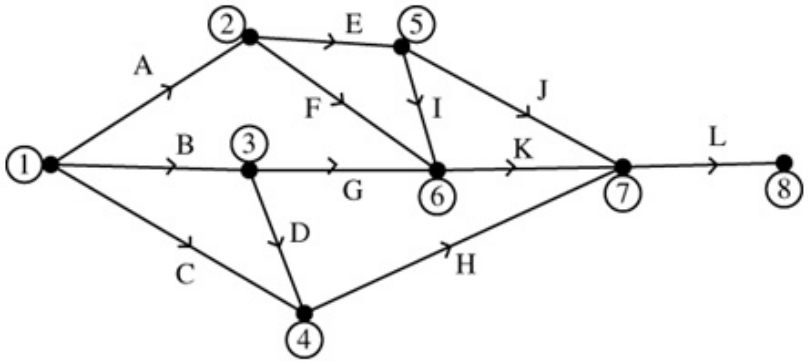
**Question:**

The precedence table for activities involved in a small project is shown below.

| Activity | Preceding activities |
|---|---|
| A | – |
| B | – |
| C | – |
| D | B |
| E | A |
| F | A |
| G | B |
| H | C, D |
| I | E |
| J | E |
| K | F, G, I |
| L | H, J, K |

Draw an activity network, using activity on edge and without using dummies, to model this project. ***E***

**Solution:**



# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 5**

**Question:**

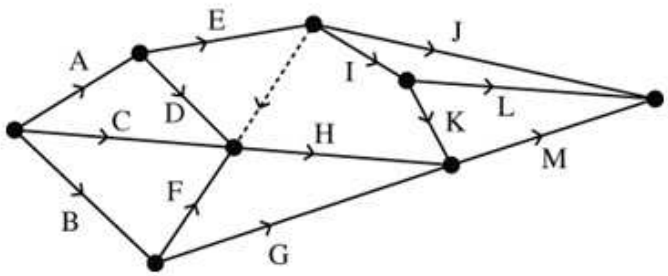
The precedence table for activities involved in manufacturing a toy is shown below.

| Activity | Preceding activities |
|---|---|
| A | – |
| B | – |
| C | – |
| D | A |
| E | A |
| F | B |
| G | B |
| H | C, D, E, F |
| I | E |
| J | E |
| K | I |
| L | I |
| M | G, H, K |

**a** Draw an activity network, using activity on arc, and exactly one dummy, to model the manufacturing process.
**b** Explain briefly why it is necessary to use a dummy in this case. ***E***

**Solution:**

**a**



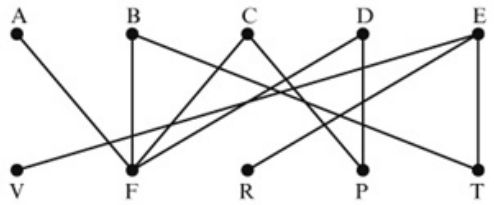

**b** For example I and J depend only on E.
H depends on C, D, E and F.

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 6**

**Question:**



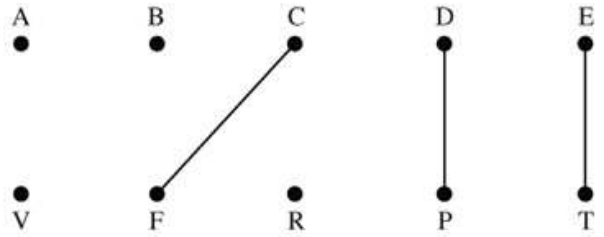

The five winners of a competition are Mr Adams (A), Mr Brown (B), Miss Church (C), Mrs Drain (D) and Ms Eagle (E). The prizes are five cars; a Vauxhall (V), a Ford (F), a Rover (R), a Peugeot (P) and a Toyota (T). The winners' preferences are summarised in the bipartite graph G shown above.

The organiser of the competition matches C with F, D with P and E with T.

**a** Indicate clearly this matching M on a matching diagram.
**b** Find an alternating path for M in G, starting at B. Use this to construct an improved matching $M'$. Show $M'$ on another diagram.
**c** Show that there is no alternating path for $M'$ in G. ***E***

**Solution:**

**a**




**b** Two possible paths – choose only one.
**i** B – T = E – V change status B = T – E = V
or
**ii** B – T = E – R change status B = T – E = R

Improved matching
**i** A = ? B = T C = F D = P E = V
or
**ii** A = ? B = T C = F D = P E = R

**c** For example, E is the only person who prefers V and the only person prefers R.
So a complete matching is not possible, so there cannot be a second alternating path.
*or*

A: F = C – P = D – F loop
A: P = D – F = C – P loop

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 7**

**Question:**

At Tesafe supermarket there are 5 trainee staff, Homan (H), Jenna (J), Mary (M), Tim (T) and Yoshie (Y). They each must spend one week in each of 5 departments, Delicatessen (D), Frozen foods (F), Groceries (G), Pet foods (P), Soft drinks (S). Next week every department requires exactly one trainee. The table below shows the departments in which the trainees have yet to spend time.

| Trainee | Departments |
|---|---|
| H | D, F, P |
| J | G, D, F |
| M | S, P, G |
| T | F, S, G |
| Y | D |

Initially H, J, M and T are allocated to the first department in their list.

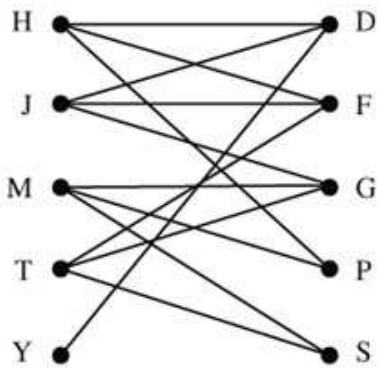
**a** Draw a bipartite graph to model this situation and indicate the initial matching in a distinctive way.

Starting from this matching,

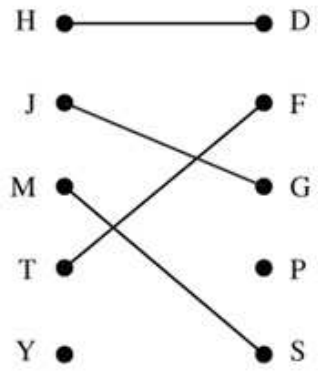
**b** use the maximum matching algorithm to find a complete matching. You must make clear your alternating path and your complete matching. ***E***

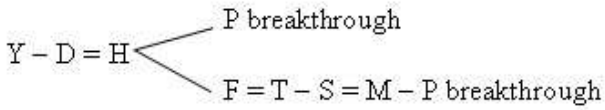
**Solution:**

**a**




**b**




Change status either

**i** Y = D – H = P giving matching H = P J = G M = S T = F Y = D

or

**ii** Y = D – H = F – T = S – M = P giving matching
H = F J = G M = P T = S Y = D

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

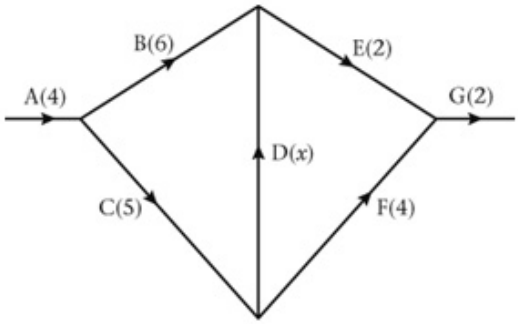
**Review Exercise 2**
**Exercise A, Question 8**

**Question:**

**a** Draw an activity network for the project described in this precedence table, using as few dummies as possible.

| Activity | Must be preceded by: |
|---|---|
| A | – |
| B | A |
| C | A |
| D | A |
| E | C |
| F | C |
| G | B, D, E, F |
| H | B, D, E, F |
| I | F, D |
| J | G, H, I |
| K | F, D |
| L | K |

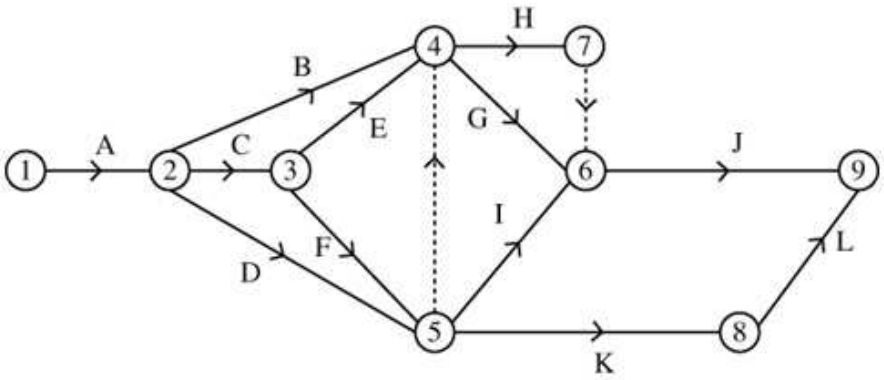
**b** A different project is represented by the activity network shown below. The duration of each activity is shown in brackets.




Find the range of values of $x$ that will make D a critical activity. ***E***

**Solution:**

**a**




**b** D will only be critical if it lies on the longest path

| Path A to G | Length |
|---|---|
| A – B – E – G | 14 |
| A – C – F – G | 15 |
| A – C – D – E – G | $13+x$ |

So we need $13+x$ to be the longest, or equal longest

$13+x \geq 15$

$x \geq 2$

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 9**

**Question:**


**a** **b**

Five members of staff 1, 2, 3, 4 and 5 are to be matched to five jobs A, B, C, D and E. A bipartite graph showing the possible matchings is given in **a** and an initial matching M is given in **b**.

There are several distinct alternating paths that can be generated from M. Two such paths are
$2-B=4-E$ and
$2-A=3-D=5-E$.

**a** Use each of these two alternating paths, in turn, to write down the complete matchings they generate.

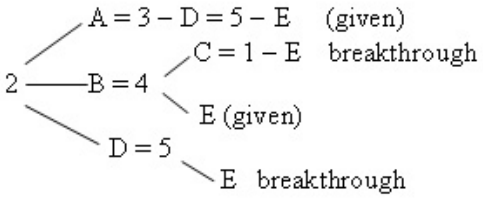
Use the maximum matching algorithm and the initial matching M,

**b** find two further distinct alternating paths, making your reasoning clear. ***E***

**Solution:**

**a** $2-B=4-E$ change status $2=B-4=E$
giving matching $1=C \quad 2=B \quad 3=A \quad 4=E \quad 5=D$
$2-A=3-D=5-E$ change status $2=A-3=D-5=E$
giving matching $1=C \quad 2=A \quad 3=D \quad 4=B \quad 5=E$

**b**




so the two further paths are: $2-B=4-C=1-E$
and $2-D=5-E$

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 10**

**Question:**

Two fertilisers are available, a liquid $X$ and a powder $Y$. A bottle of $X$ contains 5 units of chemical $A$, 2 units of chemical $B$ and $\frac{1}{2}$ unit of chemical $C$. A packet of $Y$ contains 1 unit of $A$, 2 units of $B$ and 2 units of $C$.
A professional gardener makes her own fertiliser. She requires at least 10 units of $A$, at least 12 units of $B$ and at least 6 units of $C$.
She buys $x$ bottles of $X$ and $y$ packets of $Y$.

**a** Write down the inequalities which model this situation.

**b** On the grid provided construct and label the feasible region.

A bottle of $X$ costs £2 and a packet of $Y$ costs £3.

**c** Write down an expression, in terms of $x$ and $y$, for the total cost £$T$.

**d** Using your graph, obtain the values of $x$ and $y$ that give the minimum value of $T$. Make your method clear and calculate the minimum value of $T$.

**e** Suggest how the situation might be changed so that it could no longer be represented graphically. ***E***

**Solution:**

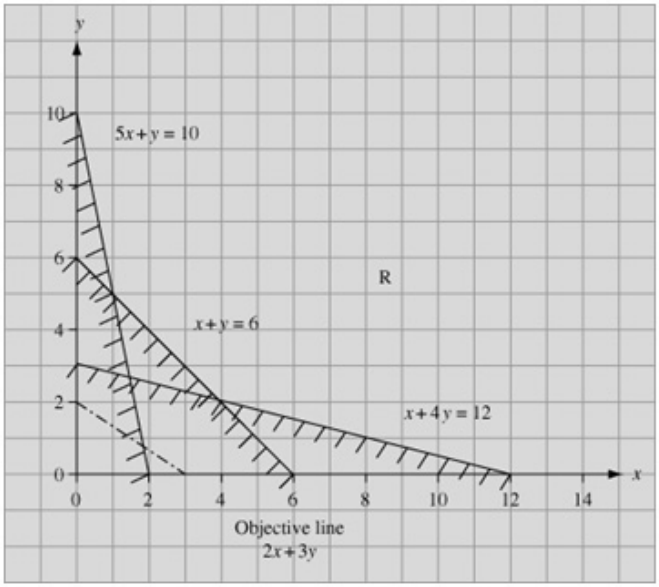
**a** Chemical A $5x+y \geq 10$

Chemical B $2x+2y \geq 12 \quad [x+y \geq 6]$

Chemical C $\frac{1}{2}x+2y \geq 6 \quad [x+4y \geq 12]$

$x, y \geq 0$

**b**




**c** $T = 2x+3y$

**d** $(4, 2)\ T = 14$

**e** If there were 3 or more variables the problem could not be solved graphically.
So adding a third fertiliser Z, would mean a graphical method could not be used.

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 11**

**Question:**

At a water sports centre there are five new instructors, Ali (A), George (G), Jo (J), Lydia (L) and Nadia (N). They are to be matched to five sports, canoeing (C), scuba diving (D), surfing (F), sailing (S) and water skiing (W).

The table indicates the sports each new instructor is qualified to teach.

| Instructor | Sport |
|---|---|
| A | C, F, W |
| G | F |
| J | D, C, S |
| L | S, W |
| N | D, F |

Initially, A, G, J and L are each matched to the first sport in their individual list.

**a** Draw a bipartite graph to model this situation and indicate the initial matching in a distinctive way.

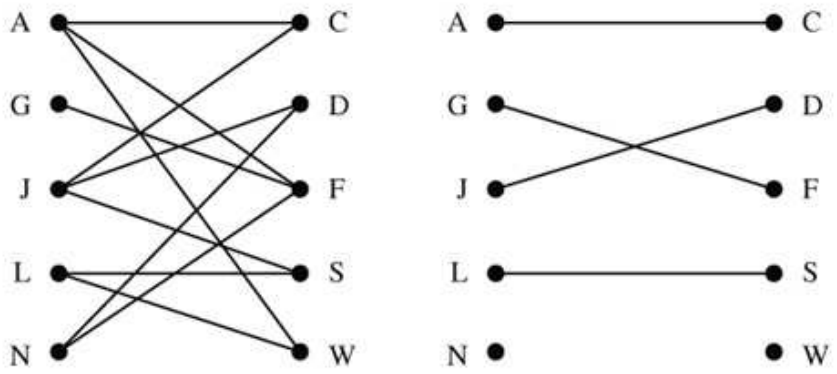
**b** Starting from this initial matching, use the maximum matching algorithm to find a complete matching. You must clearly list any alternating paths used.

Given that on a particular day J must be matched to D,

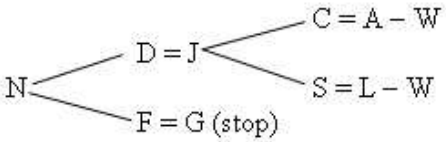
**c** explain why it is no longer possible to find a complete matching. ***E***

**Solution:**

**a**




**b**




Breakthrough, so choose just one of these.

Change status to give

$N = D - J = C - A = W$ giving matching $A = W \quad G = F \quad J = C \quad L = S \quad N = D$

$N = D - J = S - L = W$ giving matching $A = C \quad G = F \quad J = S \quad L = W \quad N = D$

**c** If J does D, N must do F leaving G without a task.

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 12**

**Question:**

A company produces two types of self-assembly wooden bedroom suites, the 'Oxford' and the 'York'. After the pieces of wood have been cut and finished, all the materials have to be packaged. The table below shows the time, in hours, needed to complete each stage of the process and the profit made, in pounds, on each type of suite.

| | Oxford | York |
|---|---|---|
| Cutting | 4 | 6 |
| Finishing | 3.5 | 4 |
| Packaging | 2 | 4 |
| Profit (£) | 300 | 500 |

The times available each week for cutting, finishing and packaging are 66, 56 and 40 hours respectively.
The company wishes to maximise its profit.
Let $x$ be the number of Oxford, and $y$ be the number of York suites made each week.

**a** Write down the objective function.

**b** In addition to

$$2x + 3y \leq 33,$$
$$x \geq 0,$$
$$y \geq 0.$$

find two further inequalities to model the company's situation.

**c** Illustrate all the inequalities, indicating clearly the feasible region.

**d** Explain how you would locate the optimal point.

**e** Determine the number of Oxford and York suites that should be made each week and the maximum profit gained.

It is noticed that when the optimal solution is adopted, the time needed for one of the three stages of the process is less than that available.

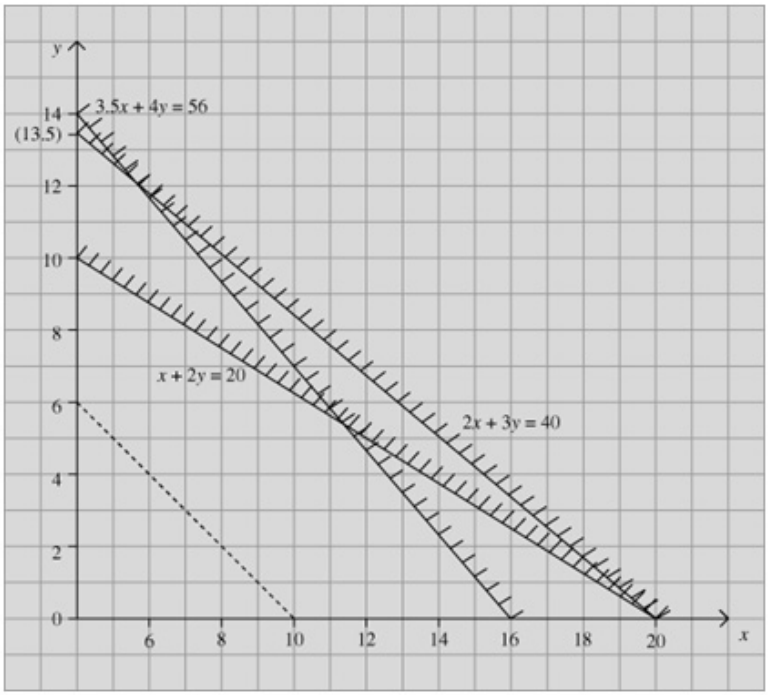
**f** Identify this stage and state by how many hours the time may be reduced. ***E***

**Solution:**

**a** Maximise $P = 300x + 500y$

**b** Finishing $3.5x + 4y \leq 56 \Rightarrow 7x + 8y \leq 112$ (o.e.)
Packing $2x + 4y \leq 40 \Rightarrow x + 2y \leq 20$ (o.e.)

**c**




**d** For example, *point testing*

- Test all corner points in feasible region.
- Find profit at each and select point yielding maximum.

*profit line*

- Draw profit lines.
- Select point on profit line furthest from the origin.

**e** Using a correct, complete method.
make 6 Oxford and 7 York profit = £5 300

$(6, 7) \rightarrow 5300$ $(14.4, 1.4) \xrightarrow{\text{integer}} (14, 1) \rightarrow 4700$ $(16, 0) \rightarrow 4800$

$(0, 10) \rightarrow 5000$

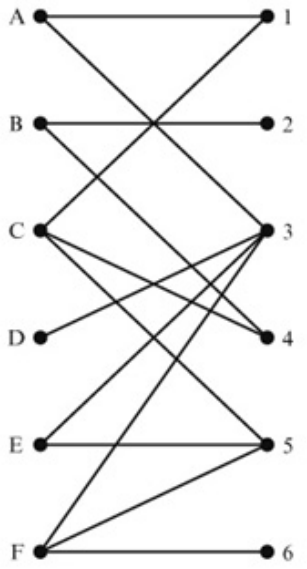
**f** The line $3.5x + 4y = 49$ passes through (6, 7) so reduce *finishing* by 7 hours.

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 13**

**Question:**



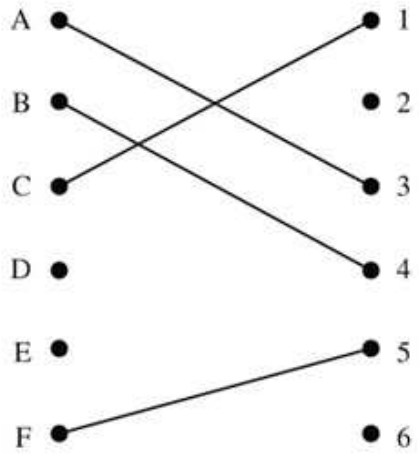

The bipartite graph above shows the possible allocations of people A, B, C, D, E and F to tasks 1, 2, 3, 4, 5 and 6.

An initial matching is obtained by matching the following pairs
A to 3, B to 4, C to 1, F to 5.

**a** Show this matching in a distinctive way on a diagram.
**b** Use an appropriate algorithm to find a maximal matching. You should state any alternating paths you have used. ***E***

**Solution:**

**a**




**b** There are six possible pairs of alternating paths – you must only choose one pair.

**i** D – 3 = A – 1 = C – 4 = B – 2 then E – 5 = F – 6
**ii** D – 3 = A – 1 = C – 5 = F – 6 then E – 5 = C – 4 = B – 2
**iii** E – 3 = A – 1 = C – 4 = B – 2 then D – 3 = E – 5 = F – 6
**iv** E – 3 = A – 1 = C – 5 = F – 6 then D – 3 = E – 5 = C – 4 = B – 2
**v** E – 5 = F – 6 then D – 3 = A – 1 = C – 4 = B – 2
**vi** E – 5 = F – 3 = A – 1 = C – 4 = B – 2 then D – 3 = F – 6

changing status, each of these give the same complete matching
A = 1 C = 4 E = 5
B = 2 D = 3 F = 6

Remember to update your 'initial' matching after the first pass through the algorithm. The first alternating path switches things around and your second path needs to take these changes into account.

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 14**

**Question:**

Six newspaper reporters Asif (A), Becky (B), Chris (C), David (D), Emma (E) and Fred (F), are to be assigned to six news stories Business (1), Crime (2), Financial (3), Foreign (4), Local (5) and Sport (6). The table shows possible allocations of reporters to news stories. For example, Chris can be assigned to any one of stories 1, 2 or 4.

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| A | | | | | √ | |
| B | √ | | | √ | | |
| C | √ | √ | | √ | | |
| D | | | | | √ | |
| E | | | √ | | √ | √ |
| F | | | | √ | | |

**a** Show these possible allocations on bipartite graph.

A possible matching is
A to 5, C to 1, E to 6, F to 4.

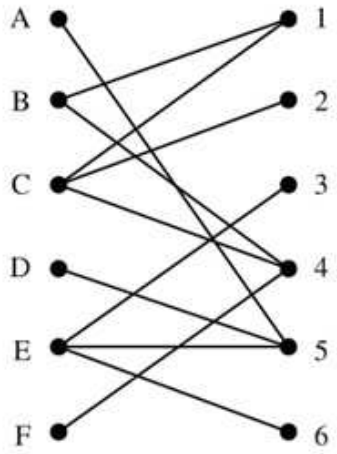
**b** Show this information, in a distinctive way, on a diagram.

**c** Use an appropriate algorithm to find a maximal matching. You should list any alternating paths you have used.

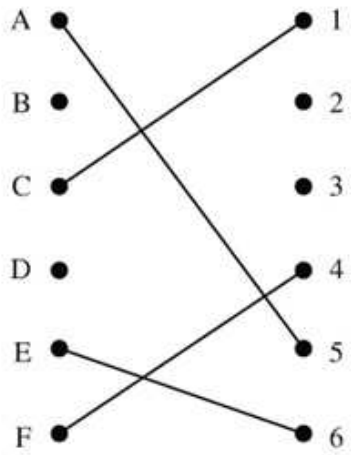
**d** Explain why it is not possible to find a complete matching. ***E***

**Solution:**

**a**




**b**




**c** B−1 = C−2 change status B = 1 − C = 2
giving improved matching A = 5 B = 1 C = 2 D = ? E = 6 F = 4

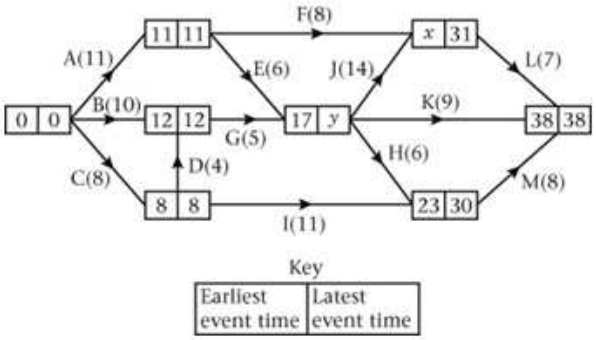
**d** For example, E is the only person who can do 3 and also the only person who can do 6, so a 1−1 complete matching is not possible.

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 15**

**Question:**




A project is modelled by the activity network in the diagram. The activities are represented by the arcs. One worker is required for each activity. The number in brackets on each arc gives the time, in hours, to complete the activity. The earliest event time and the latest event time are given by the numbers in the left box and right box respectively.

**a** State the value of $x$ and the value of $y$.
**b** List the critical activities.
**c** Explain why at least 3 workers will be needed to complete this project in 38 hours.
**d** Schedule the activities so that the project is completed in 38 hours using just 3 workers. You must make clear the start time and finish time of each activity. ***E***

**Solution:**

$x = 31$ ← highest of $(11+8)$ and $(17+14)$

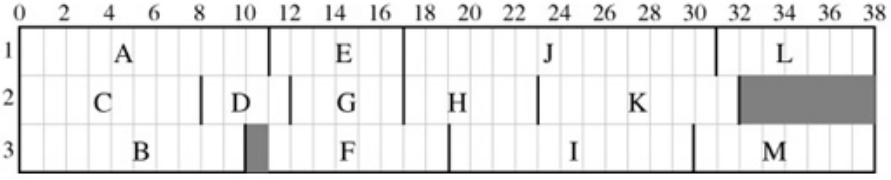
**a** $y = 17$ ← lowest of $(31-14), (38-9)$ and $(30-6)$

**b** A, E, J – L, G, C – D

All critical activities have a zero total float.

**c** $107 \div 38 = 2.8$ (1 d.p.) so at least 3 workers needed

**d** For example,



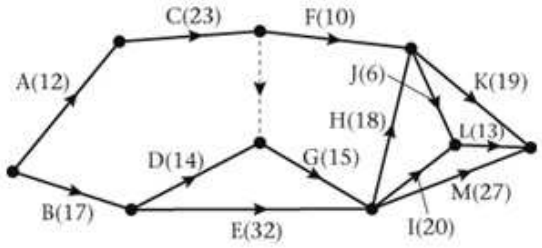
# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 16**

**Question:**

The network shows the activities involved in the process of producing a perfume. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, taken to complete the activity.




**a** Calculate the early time and the late time for each event, showing them on a diagram.

**b** Hence determine the critical activities.

**c** Calculate the total float time for D.

Each activity requires only one person.

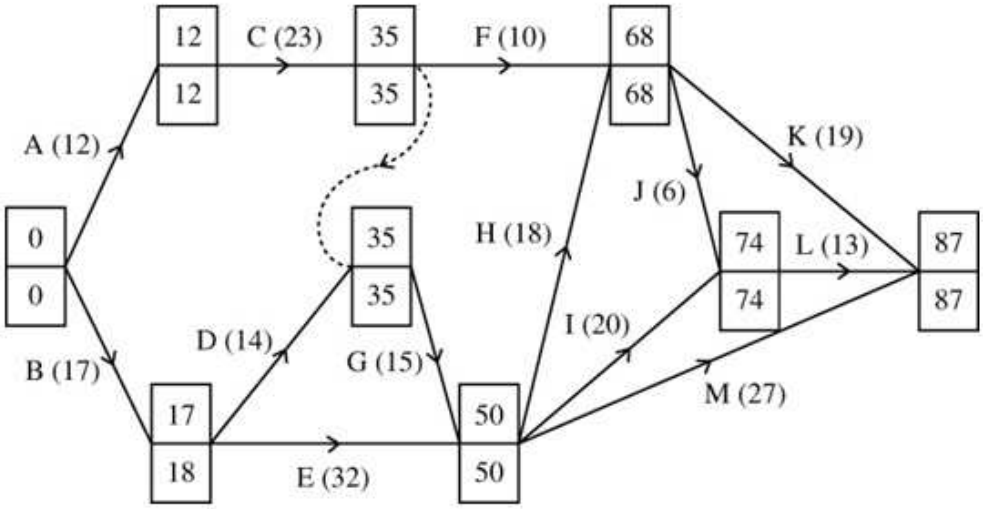
**d** Find a lower bound for the number of workers needed to complete the process in the minimum time.

Given that there are only three workers available, and that workers may **not** share an activity,

**e** schedule the activities so that the process is completed in the shortest time, use a time line. State the new shortest time. ***E***

**Solution:**

**a**



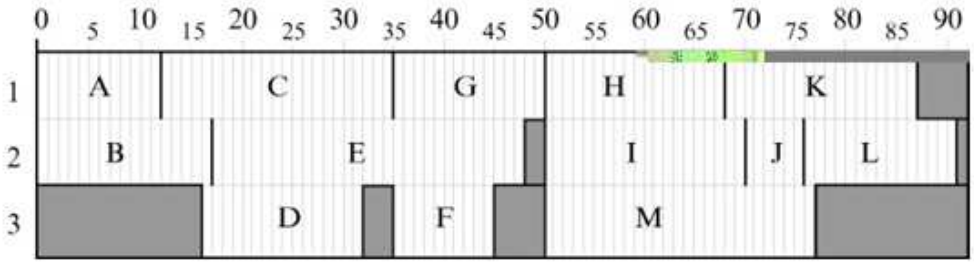

**b** A, C, E, H, J, K and L

All critical activities have a zero total float.

**c** Total float $= 35 - 17 - 14 = 4$

**d** *Either* $226 \div 87 = 2.6$ (1 d.p.) so at least 3 workers needed
*or* 69 hours into the project activities J, K, I and M *must* be happening
so at least 4 workers will be needed.

**e**




New shortest time is 89 hours

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 17**

**Question:**

The Young Enterprise Company 'Decide', is going to produce badges to sell to decision maths students. It will produce two types of badge.

Badge 1 reads 'I made the decision to do maths' and
Badge 2 reads 'Maths is the right decision'

'Decide' must produce at least 200 badges and has enough material for 500 badges. Market research suggests that the number produced of Badge 1 should be between 20% and 40% of the total number of badges made.

The company makes a profit of 30p on each Badge 1 sold and 40p on each Badge 2. It will sell all that it produces, and wishes to maximise its profit.

Let $x$ be the number produced of Badge 1 and $y$ be the number of Badge 2.

**a** Formulate this situation as a linear programming problem, simplifying your inequalities so that all the coefficients are integers.

**b** On suitable axes, construct and clearly label the feasible region.

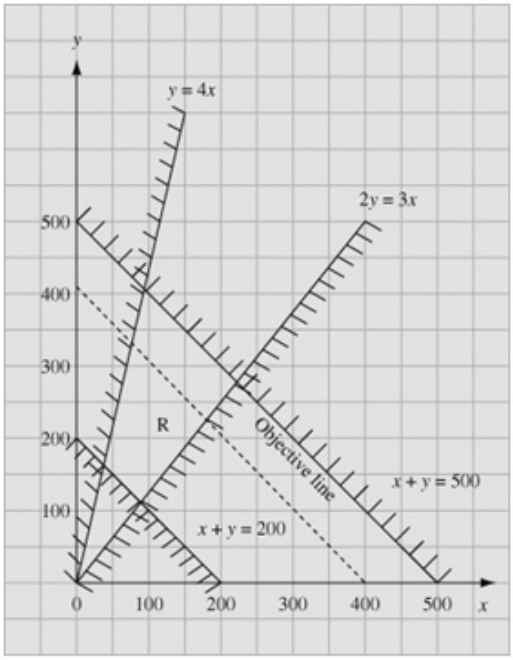
**c** Using your graph, advise the company on the numbers of each badge it should produce. State the maximum profit 'Decide' will make. ***E***

**Solution:**

**a** Objective: maximise $P = 30x + 40y$ (or $P = 0.3x + 0.4y$)
subject to:

$$x + y \geq 200$$
$$x + y \leq 500$$
$$x \geq \frac{20}{100}(x+y) \Rightarrow 4x \geq y$$
$$x \leq \frac{40}{100}(x+y) \Rightarrow 3x \leq 2y$$
$$x, y \geq 0$$

**b**




**c** Visible use of objective line method – objective line drawn or vertex testing – all 4 vertices tested

Vertex testing

$(40,160) \rightarrow 7600$
$(80,120) \rightarrow 7200$
$(100,400) \rightarrow 19\,000$
$(200,300) \rightarrow 18\,000$

Intersection of $y = 4x$ and $x + y = 500$
(100, 400) profit £190 (or 19 000 p)

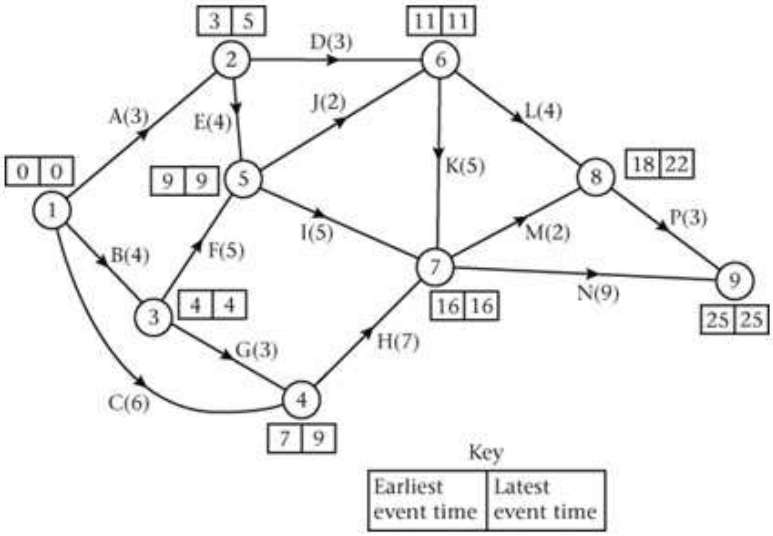
# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 18**

**Question:**

A building project is modelled by the activity network shown. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, taken to complete the activity. The left box entry at each vertex is the earliest event time and the right box entry is the latest event time




**a** Determine the critical activities and state the length of the critical path.

**b** State the total float for each non-critical activity.

**c** On a grid, draw a cascade (Gantt) chart for the project.

Given that each activity requires one worker,

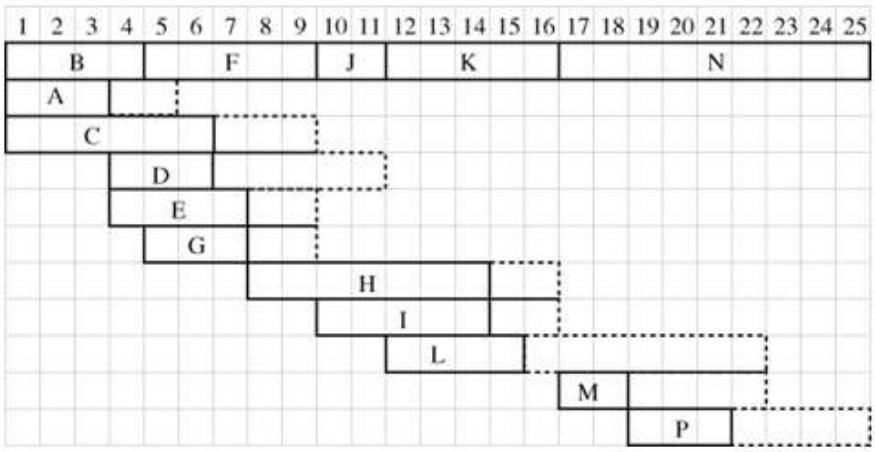
**d** draw up a schedule to determine the minimum number of workers required to complete the project in the critical time. State the minimum number of workers. ***E***

**Solution:**

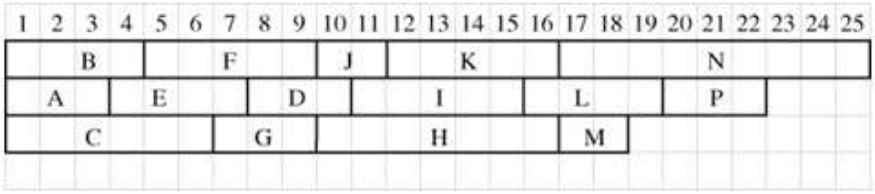
**a** Critical activities are B, F, J, K and N length of critical path is 25 hours
I is not critical.

**b** Total float on A = 5 − 0 − 3 = 2
Total float on C = 9 − 0 − 6 = 3
Total float on D = 11 − 3 − 3 = 5
Total float on E = 9 − 3 − 4 = 2
Total float on G = 9 − 4 − 3 = 2
Total float on H = 16 − 7 − 7 = 2
Total float on I = 16 − 9 − 5 = 2
Total float on L = 22 − 11 − 4 = 7
Total float on M = 22 − 16 − 2 = 4
Total float on P = 25 − 18 − 3 = 4

**c**




**d** For example



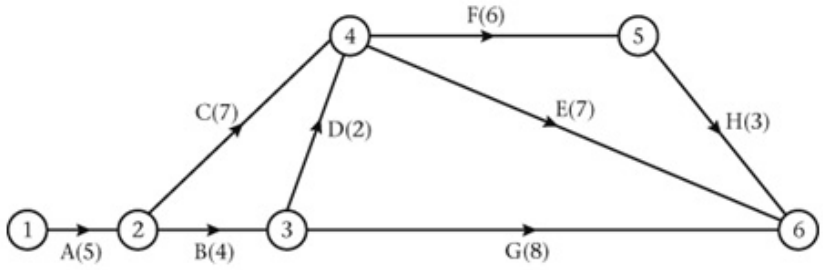

Minimum number of workers is 3.

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 19**

**Question:**




A project is modelled by the activity network shown above. The activities are represented by the edges. The number in brackets on each edge gives the time, in days, taken to complete the activity.

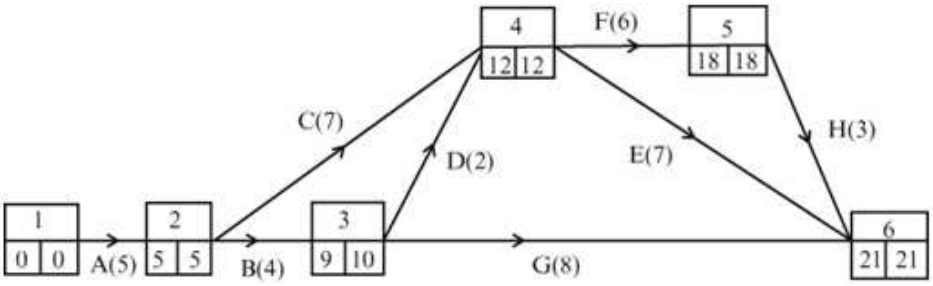
**a** Calculate the early time and the late time for each event. Write these in the boxes on the answer sheet.
**b** Hence determine the critical activities and the length of the critical path.
**c** Obtain the total float for each of the non-critical activities.
**d** Draw a cascade (Gantt) chart showing the information obtained in parts **b** and **c**.

Each activity requires one worker. Only two workers are available.

**e** Draw up a schedule and find the minimum time in which the 2 workers can complete the project. ***E***

**Solution:**

**a**

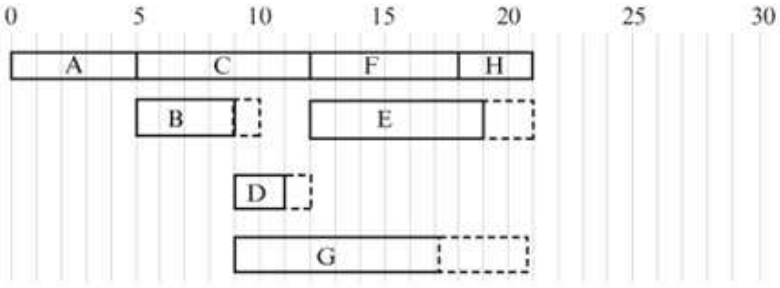



**b** Critical activities: A, C, F and H; length of critical path = 21

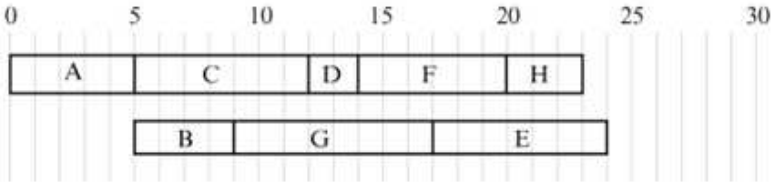
**c** Total float on B = 10 − 5 − 4 = 1 Total float on E = 21 − 12 − 7 = 2

Total float on D = 12 − 9 − 2 = 1 Total float on G = 21 − 9 − 8 = 4

**d**




**e** For example




Minimum time for 2 workers is 24 days.

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 20**

**Question:**

Becky's bird food company makes two types of bird food. One type is for bird feeders and the other for bird tables. Let $x$ represent the quantity of food made for bird feeders and $y$ represent the quantity of food made for bird tables. Due to restrictions in the production process, and known demand, the following constraints apply.

$$x+y \leq 12,$$
$$y < 2x,$$
$$2y \geq 7,$$
$$y+3x \geq 15.$$

**a** Show these constraints on a diagram and label the feasible region R.

The objective is to minimise $C = 2x+5y$.

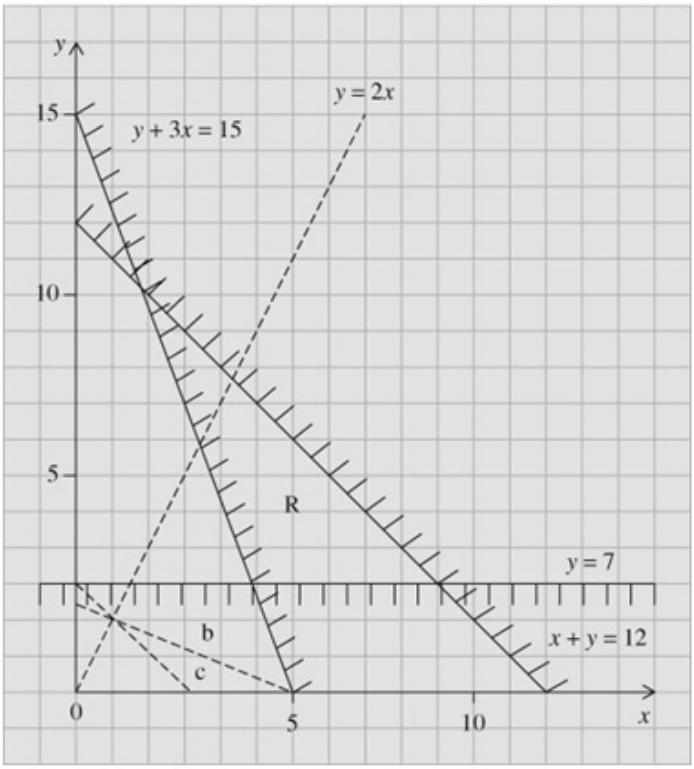
**b** Solve this problem, making your method clear. Give, as fractions, the value of $C$ and the amount of each type of food that should be produced.

Another objective (for the same constraints given above) is to maximise $P = 3x+2y$, where the variables must take integer values.

**c** Solve this problem, making your method clear. State the value of $P$ and the amount of each type of food that should be produced. ***E***

**Solution:**

**a**




**b** Visible use of objective line method – objective line drawn or vertex testing.

$\left[\left(3\frac{5}{6}, 3\frac{1}{2}\right) \rightarrow 25\frac{1}{6}\left(8\frac{1}{2}, 3\frac{1}{2}\right) \rightarrow 34\frac{1}{2}(4, 8) \rightarrow 48\,(3, 6) \rightarrow 36\right]$

Optimal point $\left(3\frac{5}{6}, 3\frac{1}{2}\right)$ with value $25\frac{1}{6}$

**c** Visible use of objective line method – objective line drawn, or vertex testing – all 4 vertices tested.

$\left(3\frac{5}{6}, 3\frac{1}{2}\right)$ not an integer try $(4,4) \rightarrow 20 \quad (4,8) \rightarrow 28$

$\left(8\frac{1}{2}, 3\frac{1}{2}\right)$ not an integer try $(8,4) \rightarrow 32 \quad (3,6) \rightarrow 21$

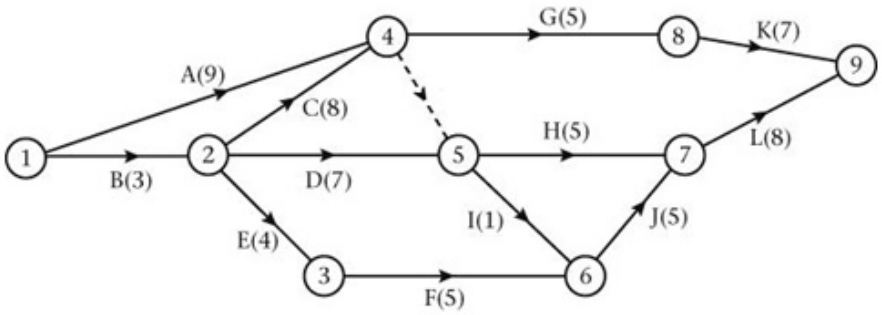
Optimal point (8, 4) with value 32

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 21**

**Question:**



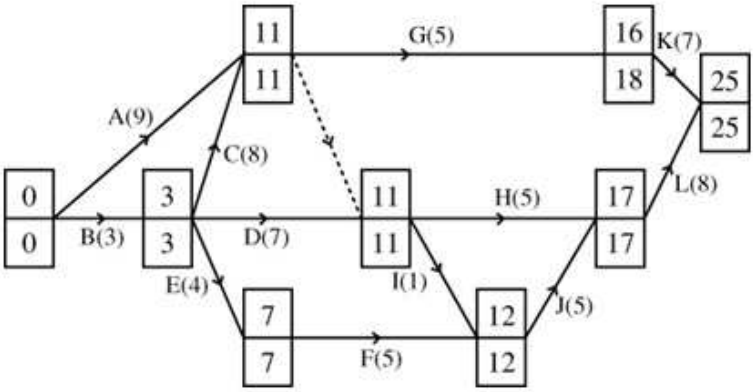

A project is modelled by the activity network shown above. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, to complete the activity. The numbers in circles give the event numbers. Each activity requires one worker.

**a** Explain the purpose of the dotted line from event 4 to event 5.
**b** Calculate the early time and the late time for each event.
**c** Determine the critical activities.
**d** Obtain the total float for each of the non-critical activities.
**e** On a grid, draw a cascade (Gantt) chart, showing the answers to parts **c** and **d**.
**f** Determine the minimum number of workers needed to complete the project in the minimum time. Make your reasoning clear. ***E***

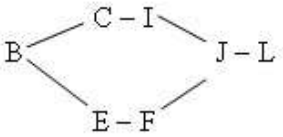
**Solution:**

**a** For example, it shows dependence but it is not an activity. G depends on A and C *only* but H and I depend on A, C *and* D.

**b**



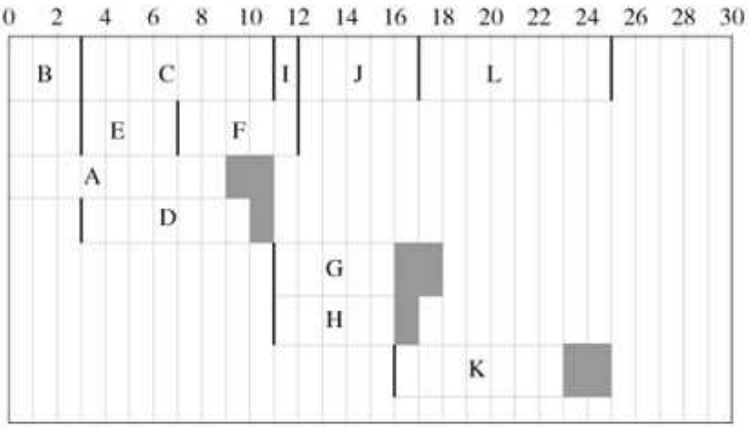

**c**




So B, C, E, F, I, J and L

**d** Total float on A = 11 − 0 − 9 = 2 Total float on H = 17 − 11 − 5 = 1
Total float on D = 11 − 3 − 7 = 1 Total float on K = 25 − 16 − 7 = 2
Total float on G = 18 − 11 − 5 = 2

**e**




**f** Arithmetic lower bound $= \frac{67}{25} = 2.68$ so a minimum of 3 workers needed.

From Gantt chart: At time 8 activities C, F, A and D must be happening, so a minimum of 4 workers are needed.

We need to take the higher of these as our best lower bound and state that a minimum of 4 workers are needed.

# Solutionbank D1

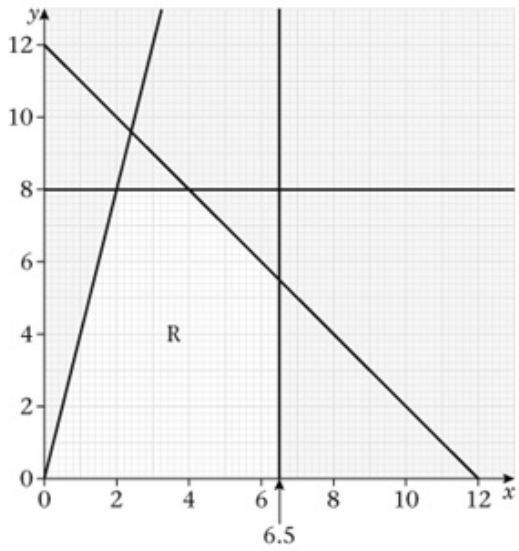
## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 22**

**Question:**

The company EXYCEL makes two types of battery, X and Y. Machinery, workforce and predicted sales determine the number of batteries EXYCEL make. The company decides to use a graphical method to find its optimal daily production of X and Y.

The constraints are modelled in the diagram. The feasible region, R, is indicated.




$x$ = the number (in thousands) of type X batteries produced each day,
$y$ = the number (in thousands) of type Y batteries produced each day.
The profit of each type X battery is 40p and on each type Y battery is 20p.
The company wishes to maximise its daily profit.

**a** Write this as a linear programming problem, in terms of $x$ and $y$, stating the objective function and all the constraints.
**b** Find the optimal number of batteries to be made each day. Show your method clearly.
**c** Find the daily profit, in £, made by EXYCEL. ***E***

**Solution:**

**a** Objective: maximise $P = 0.4x + 0.2y \quad (P = 40x + 20y)$
subject to:

$$x \leq 6.5$$
$$y \leq 8$$
$$x + y \leq 12$$
$$y \leq 4x$$
$$x, y \geq 0$$

**b** Visible use of objective line method – objective line drawn (e.g. from (2, 0) to (0, 4)) or all 5 points tested.

vertex testing
$[(0,0) \rightarrow 0; (2,8) \rightarrow 2.4; (4,8) \rightarrow 3.2; (6.5,5.5) \rightarrow 3.7; (6.5,0) \rightarrow 2.6]$

Optimal point is $(6.5,5.5) \Rightarrow 6\,500$ type $X$ and 5500 type $Y$

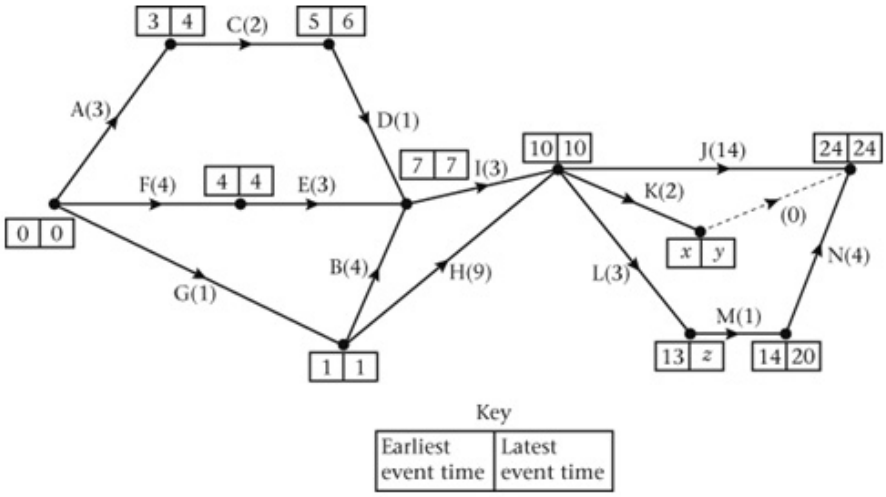
**c** $P = 0.4(6500) + 0.2(5500) = £3700$

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 23**

**Question:**




The network above shows activities that need to be undertaken in order to complete a project. Each activity is represented by an arc. The number in brackets is the duration of the activity in hours. The early and late event times are shown at each node. The project can be completed in 24 hours.

**a** Find the values of $x$, $y$ and $z$.

**b** Explain the use of the dummy activity in the diagram.

**c** List the critical activities.

**d** Explain what effect a delay of one hour to activity B would have on the time taken to complete the whole project.

The company which is to undertake this project has only two full time workers available. The project must be completed in 24 hours and in order to achieve this, the company is prepared to hire additional workers at a cost of £28 per hour. The company wishes to minimise the money spent on additional workers. Any worker can undertake any task and each task requires only one worker.

**e** Explain why the company will have to hire additional workers in order to complete the project in 24 hours.

**f** Schedule the tasks to workers so that the project is completed in 24 hours and at minimum cost to the company.

**g** State the minimum extra cost to the company. ***E***

**Solution:**

**a** $x = 12 \quad y = 24 \quad z = 19$

**b** Allows J and K to be uniquely expressed in terms of their end events.

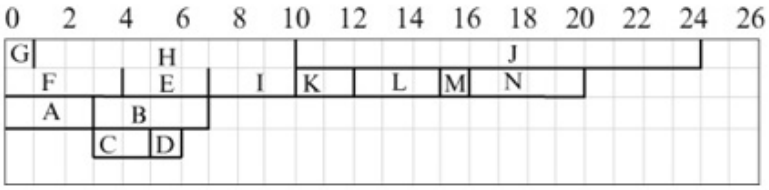
**c** F, E, I, J, G, H

**d** It would have no effect, B has a total float of 2 so a delay of one hour would still permit the project to be completed on time.

**e** For example,

- the total of activities is 54. 2 workers working for 24 hours would not be sufficient,
- $54 \div 24 = 2.25$, so 2 workers are not enough,
- 7 hours into the project A, B, C, D, E, F and G must be completed; these activities require 18 hours to complete them so 2 workers could not be enough.

**f**



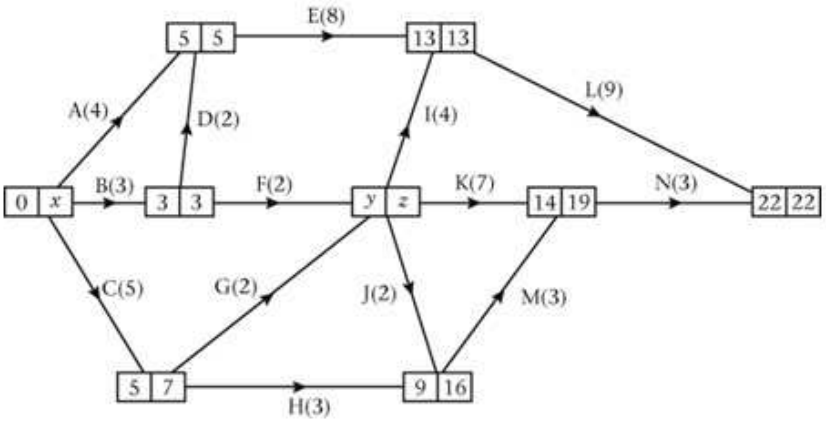

**g** 10 extra hours (7 + 3) so £280

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 24**

**Question:**




A trainee at a building company is using critical path analysis to help plan a project. The diagram shows the trainee's activity network. Each activity is represented by an arc and the number in brackets on each arc is the duration of the activity, in hours.

**a** Find the values of $x$, $y$ and $z$.

**b** State the total length of the project and list the critical activities.

**c** Calculate the total float time on

**i** activity N,

**ii** activity H.

The trainee's activity network is checked by the supervisor who finds a number of errors and omissions in the diagram. The project should be represented by the following precedence table.

| Activity | Must be preceded by: | Duration |
|---|---|---|
| A | – | 4 |
| B | – | 3 |
| C | – | 5 |
| D | B | 2 |
| E | A, D | 8 |
| F | B | 2 |
| G | C | 2 |
| H | C | 3 |
| I | F, G | 4 |
| J | F, G | 2 |
| K | F, G | 7 |
| L | E, I | 9 |
| M | H, J | 3 |
| N | E, I, K, M | 3 |
| P | E, I | 6 |
| Q | H, J | 5 |
| R | Q | 7 |

**d** By adding activities and dummies amend the diagram so that it represents this precedence table.

**e** Find the total time needed to complete this project. ***E***

**Solution:**

**a** $x = 0$

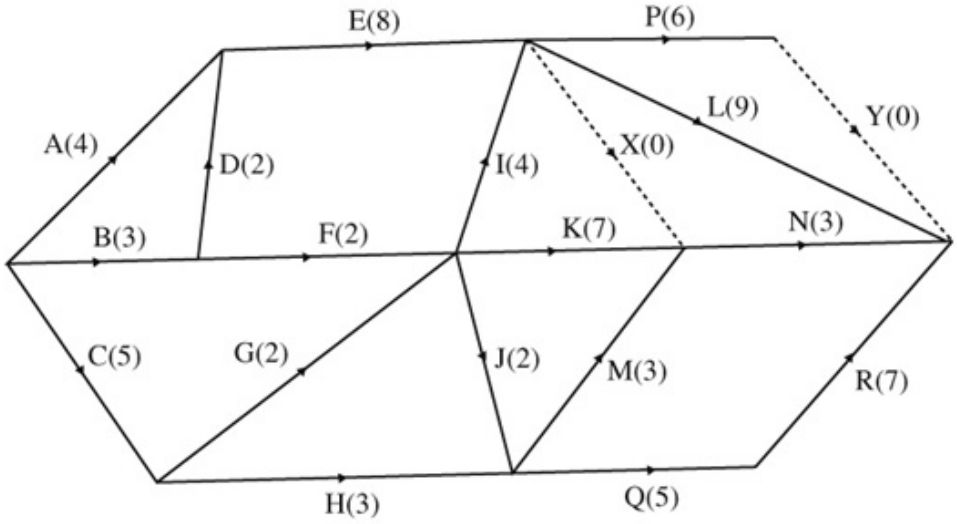
$y = 7$ [latest out of $(3+2)$ and $(5+2)$]

$z = 9$ [Earliest out of $(13-4)$ and $(19-7)$ and $(16-2)$]

**b** Length is 22
Critical activities: B, D, E and L

**c** Total float on N $= 22-14-3=5$
Total float on H $= 16-5-3=8$

**d**




**e** 22 hours [critical path is still B–D–E–L]

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 25**

**Question:**

A chemical company produces two products $X$ and $Y$. Based on potential demand, the total production each week must be at least 380 gallons. A major customer's weekly order for 125 gallons of $Y$ must be satisfied.

Product $X$ requires 2 hours of processing time for each gallon and product $Y$ requires 4 hours of processing time for each gallon. There are 1200 hours of processing time available each week. Let $x$ be the number of gallons of $X$ produced and $y$ be the number of gallons of $Y$ produced each week.

**a** Write down the inequalities which $x$ and $y$ must satisfy.

It costs £3 to produce 1 gallon of $X$ and £2 to produce 1 gallon of $Y$. Given that the total cost of production is £$C$,

**b** express $C$ in terms of $x$ and $y$.

The company wishes to minimise the total cost.

**c** Using a graphical method, solve the resulting linear programming problem. Find the optimal values of $x$ and $y$ and the resulting total cost.

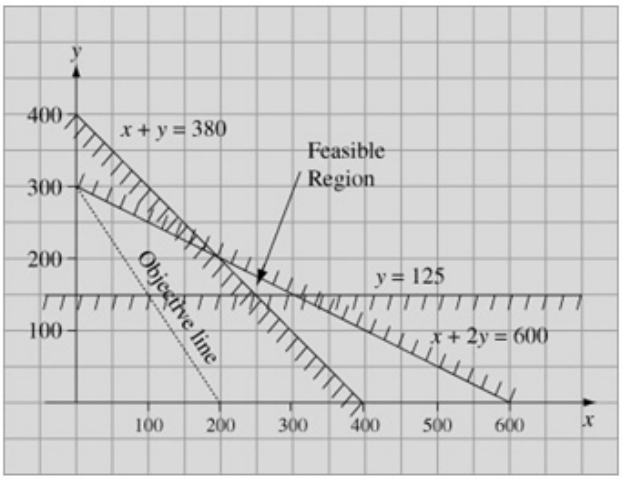
**d** Find the maximum cost of production for all possible choices of $x$ and $y$ which satisfy the inequalities you wrote down in part **a**. ***E***

**Solution:**

**a** $x+y \geq 380$ (total production is at least 380)
$y \geq 125$ (at least 125 gallons of $y$)
$2x+4y \leq 1200 \Rightarrow x+2y \leq 600$ (processing time)
$x \geq 0$

**b** $C = 3x+2y$

**c**




Visible use of objective line method – objective line drawn or vertex testing – all 3 vertices [(160, 220) → 920; (255, 125) → 1015; (350, 125) → 1300]
Optimal point is (160, 220) value £920.

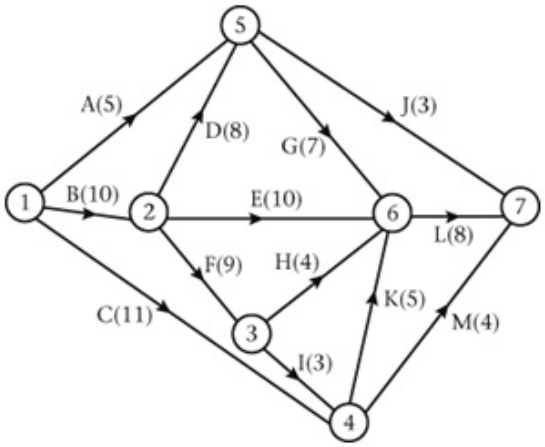
**d** (350, 125) $C =$ £1300

# Solutionbank D1

## Edexcel AS and A Level Modular Mathematics

**Review Exercise 2**
**Exercise A, Question 26**

**Question:**




The diagram shows the activity network used to model a building project. The activities are represented by the edges. The number in brackets on each edge represents the time, in days, to complete the activity.

**a** Calculate the early time and the late time for each event.

**b** Calculate the total float for each activity.

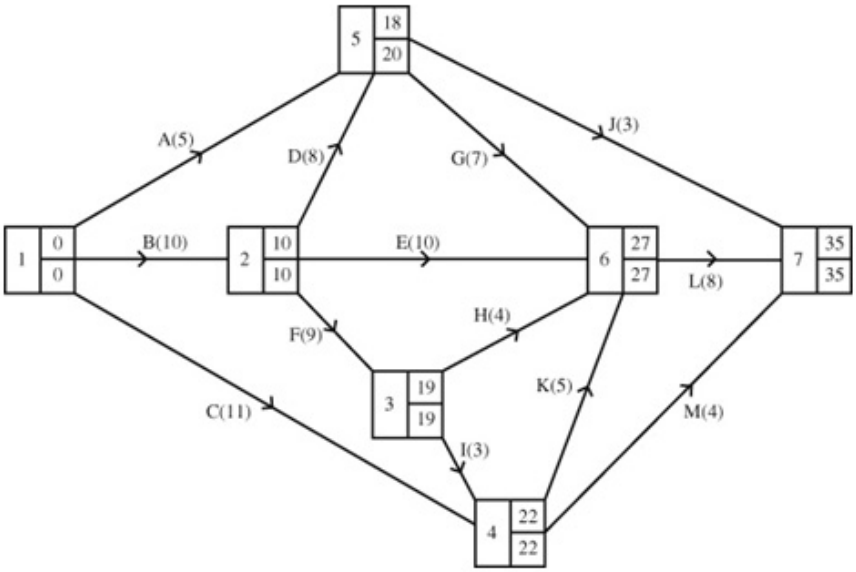
**c** Hence determine the critical activities. Write down the length of the critical path.

Owing to the breakdown of a piece of equipment the time taken to complete activity H increases to 9 days.

**d** Obtain the new critical path and its length. ***E***

**Solution:**

**a**




**b** Total float on A = 20 − 0 − 5 = 15
Total float on B = 10 − 0 − 10 = 0
Total float on C = 22 − 0 − 11 = 11
Total float on D = 20 − 10 − 8 = 2
Total float on E = 27 − 10 − 10 = 7
Total float on F = 19 − 10 − 9 = 0
Total float on G = 27 − 18 − 7 = 2
Total float on H = 27 − 19 − 4 = 4
Total float on I = 22 − 19 − 3 = 0
Total float on J = 35 − 18 − 3 = 14
Total float on K = 27 − 22 − 5 = 0
Total float on L = 35 − 27 − 8 = 0
Total float on M = 35 − 22 − 4 = 9

**c** Critical activities: B, F, I, K and L
length of critical path is 35 days

**d** New critical path is B–F–H–L
length of new critical path is 36 days