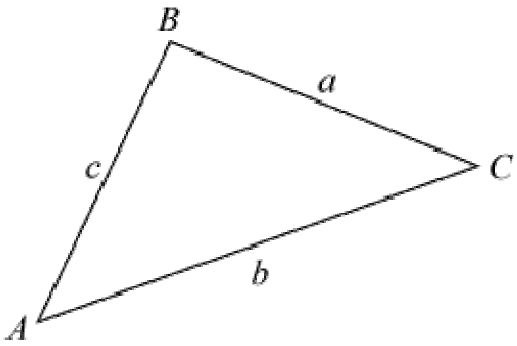
Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

The sine and cosine rule
Exercise A, Question 1

Question:

In each of parts (a) to (d), given values refer to the general triangle:



(a) Given that $a = 8$ cm, $A = 30°$, $B = 72°$, find b.

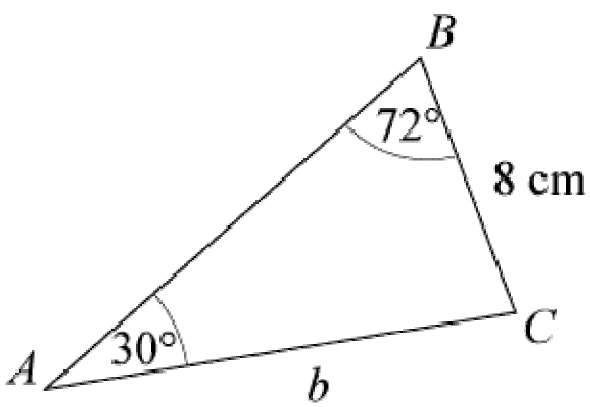
(b) Given that $a = 24$ cm, $A = 110°$, $C = 22°$, find c.

(c) Given that $b = 14.7$ cm, $A = 30°$, $C = 95°$, find a.

(d) Given that $c = 9.8$ cm, $B = 68.4°$, $C = 83.7°$, find a.

Solution:

(a)



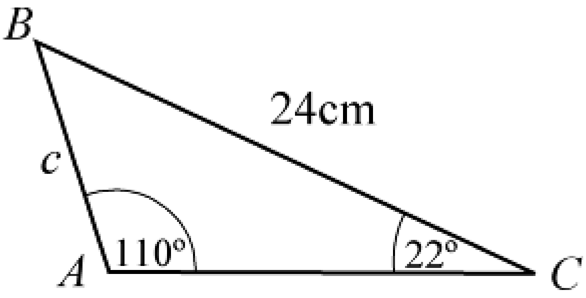
Using $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{b}{\sin 72°} = \frac{8}{\sin 30°}$

$\Rightarrow \quad b = \frac{8 \sin 72°}{\sin 30°} = 15.2$ cm (3 s.f.)

(Check: as $72° > 30°$, $b > 8$ cm.)

(b)



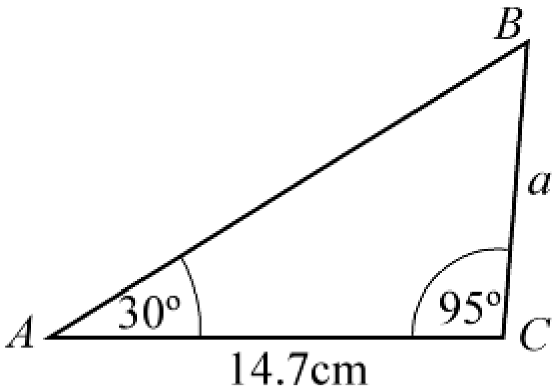
Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{c}{\sin 22°} = \frac{24}{\sin 110°}$

$\Rightarrow \quad c = \frac{24 \sin 22°}{\sin 110°}$ = 9.57 cm (3 s.f.)

(As 110° > 22°, 24cm > *c*.)

(c)



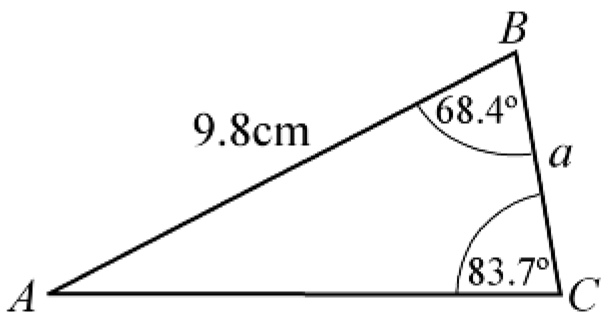
$\angle$ ABC = 180° − (30 + 95)° = 55°

Using $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{a}{\sin 30°} = \frac{14.7}{\sin 55°}$

$\Rightarrow \quad a = \frac{14.7 \sin 30°}{\sin 55°}$ = 8.97 cm (3 s.f.)

(d)



$\angle$ BAC = 180° − (68.4 + 83.7)° = 27.9°

Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\frac{a}{\sin 27.9°} = \frac{9.8}{\sin 83.7°}$$

$$\Rightarrow \quad a = \frac{9.8 \sin 27.9°}{\sin 83.7°} = 4.61 \text{ cm (3 s.f.)}$$

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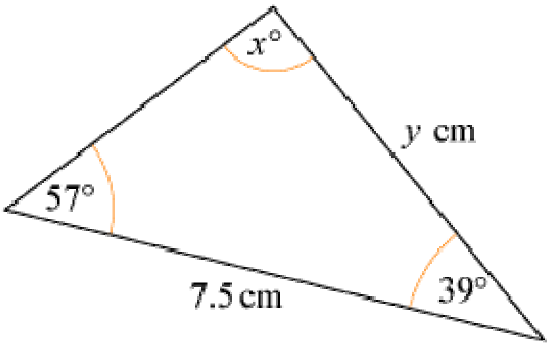
The sine and cosine rule
Exercise A, Question 2

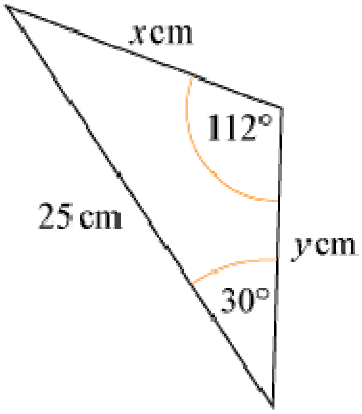
Question:

In each of the following triangles calculate the values of *x* and *y*.

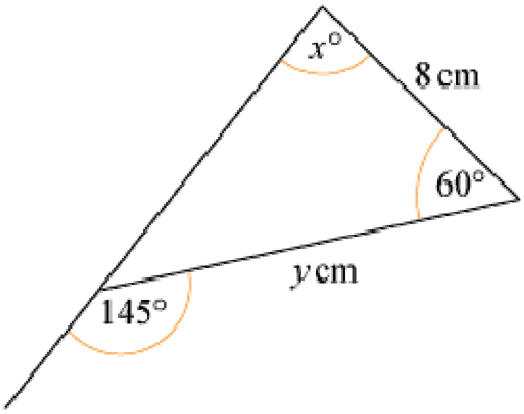
(a)



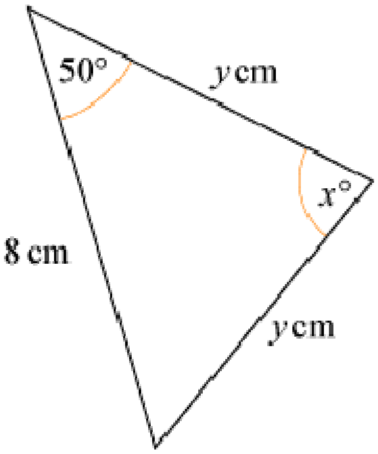
(b)



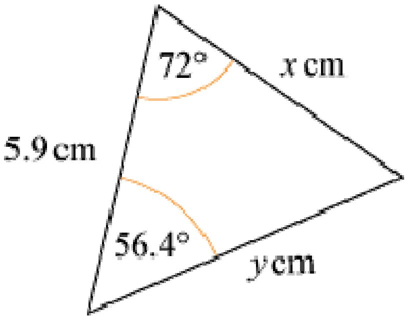
(c)



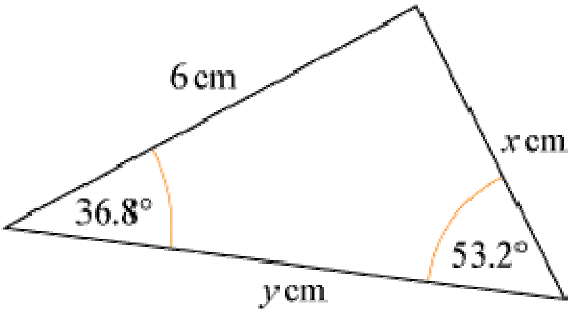
(d)



(e)

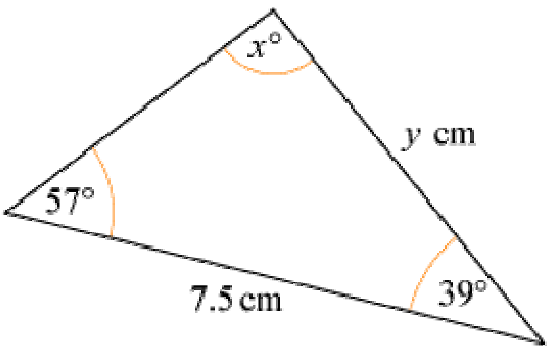


(f)



Solution:

(a)



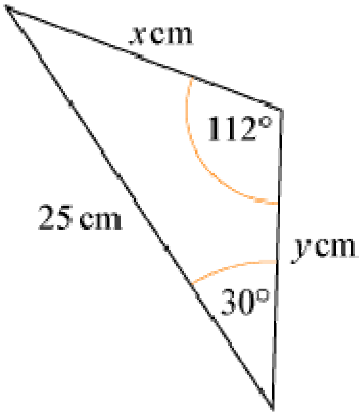
$x = 180 - (57 + 39) = 84$

Using $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\Rightarrow \quad \frac{y}{\sin 57°} = \frac{7.5}{\sin 84°}$

$\Rightarrow \quad y = \frac{7.5 \sin 57°}{\sin 84°} = 6.32$ (3 s.f.)

(b)



Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\Rightarrow \quad \frac{x}{\sin 30°} = \frac{25}{\sin 112°}$

$\Rightarrow \quad x = \frac{25 \sin 30°}{\sin 112°} = 13.5$ (3 s.f.)

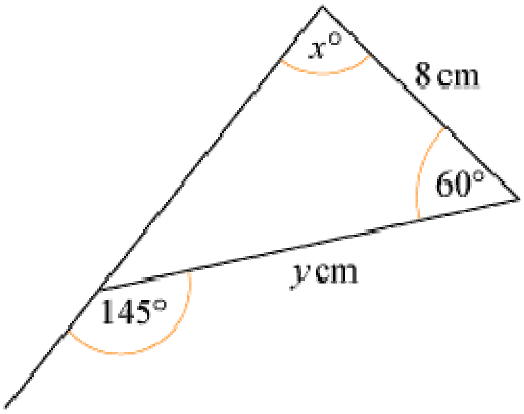
$\angle B = 180° - (112 + 30)° = 38°$

Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \quad \frac{y}{\sin 38°} = \frac{25}{\sin 112°}$

$\Rightarrow \quad y = \frac{25 \sin 38°}{\sin 112°} = 16.6$ (3 s.f.)

(c)



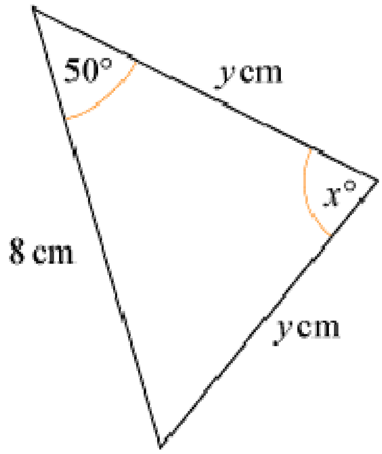
$x = 180 - (60 + 35) = 85$

Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{y}{\sin 85°} = \frac{8}{\sin 35°}$

$\Rightarrow \quad y = \frac{8 \sin 85°}{\sin 35°} = 13.9$ (3 s.f.)

(d)



$x = 180 - (50 + 50) = 80$

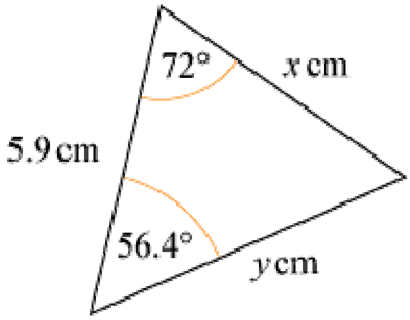
Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\Rightarrow \quad \frac{y}{\sin 50°} = \frac{8}{\sin 80°}$

$\Rightarrow \quad y = \frac{8 \sin 50°}{\sin 80°} = 6.22$ (3 s.f.)

(Note: You could use the line of symmetry to split the triangle into two right-angled triangles and use $\cos 50° = \frac{4}{y}$.)

(e)



$\angle C = 180° - (56.4 + 72)° = 51.6°$

Using $\frac{a}{\sin A} = \frac{C}{\sin C}$

$\Rightarrow \quad \frac{x}{\sin 56.4°} = \frac{5.9}{\sin 51.6°}$

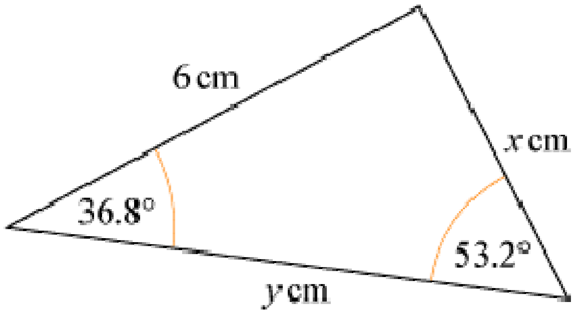
$\Rightarrow \quad x = \frac{5.9 \sin 56.4°}{\sin 51.6°} = 6.27$ (3 s.f.)

Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \quad \frac{y}{\sin 72°} = \frac{5.9}{\sin 51.6°}$

$\Rightarrow \quad y = \frac{5.9 \sin 72°}{\sin 51.6°} = 7.16$ (3 s.f.)

(f)



Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\Rightarrow \quad \frac{x}{\sin 36.8°} = \frac{6}{\sin 53.2°}$

$\Rightarrow \quad x = \frac{6 \sin 36.8°}{\sin 53.2°} = 4.49$ (3 s.f.)

$\angle B = 180° - (36.8 + 53.2)° = 90°$

Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \quad \frac{6}{\sin 53.2°} = \frac{y}{\sin 90°}$

$\Rightarrow \quad y = \frac{6 \sin 90°}{\sin 53.2°} = 7.49$ (3 s.f.)

(Note: The third angle is 90° so you could solve the problem using sine and cosine; the sine rule is not necessary.)

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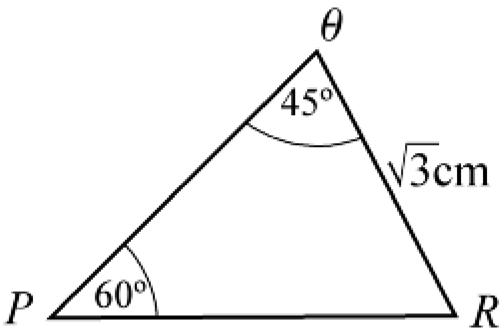
Edexcel Modular Mathematics for AS and A-Level

The sine and cosine rule
Exercise A, Question 3

Question:

In $\triangle PQR$, QR = $\sqrt{3}$ cm, $\angle$ PQR = 45° and $\angle$ QPR = 60°. Find (a) *PR* and (b) *PQ*.

Solution:



(a) Using $\frac{q}{\sin Q} = \frac{p}{\sin P}$

$\Rightarrow \quad \frac{PR}{\sin 45°} = \frac{\sqrt{3}}{\sin 60°}$

$\Rightarrow \quad PR = \frac{\sqrt{3} \sin 45°}{\sin 60°} = 1.41$ cm (3 s.f.)

(The exact answer is $\sqrt{2}$ cm.)

(b) Using $\frac{r}{\sin R} = \frac{p}{\sin P}$

$\Rightarrow \quad \frac{PQ}{\sin 75°} = \frac{\sqrt{3}}{\sin 60°}$ [Angle $R = 180° - (60 + 45)° = 75°$]

$\Rightarrow \quad PQ = \frac{\sqrt{3} \sin 75°}{\sin 60°} = 1.93$ cm (3 s.f.)

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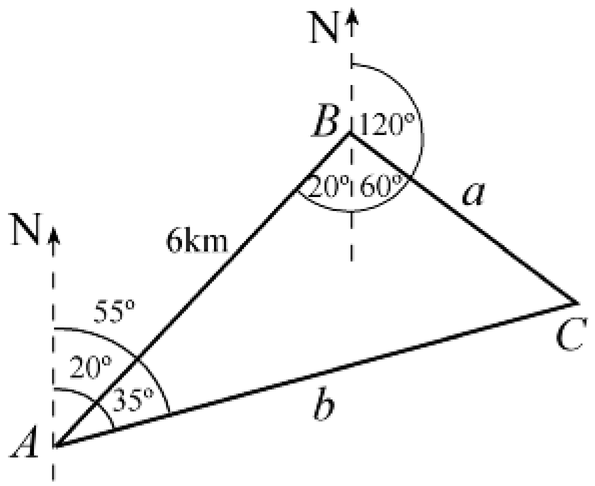
The sine and cosine rule
Exercise A, Question 4

Question:

Town B is 6 km, on a bearing of 020°, from town A. Town C is located on a bearing of 055° from town A and on a bearing of 120° from town B. Work out the distance of town C from (a) town A and (b) town B.

Solution:



$\angle$ BAC = 55 ° − 20 ° = 35 °

$\angle$ ABC = 20 ° ('Z' angles) + 60 ° (angles on a straight line) = 80 °

$\angle$ ACB = 180 ° − (80 + 35) ° = 65 °

(a) Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \frac{\text{AC}}{\sin 80°} = \frac{6}{\sin 65°}$

$\Rightarrow \text{AC} = \frac{6 \sin 80°}{\sin 65°}$ = 6.52 km (3 s.f.)

(b) Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\Rightarrow \frac{\text{BC}}{\sin 35°} = \frac{6}{\sin 65°} \Rightarrow \text{BC} = \frac{6 \sin 35°}{\sin 65°}$ = 3.80 km (3 s.f.)

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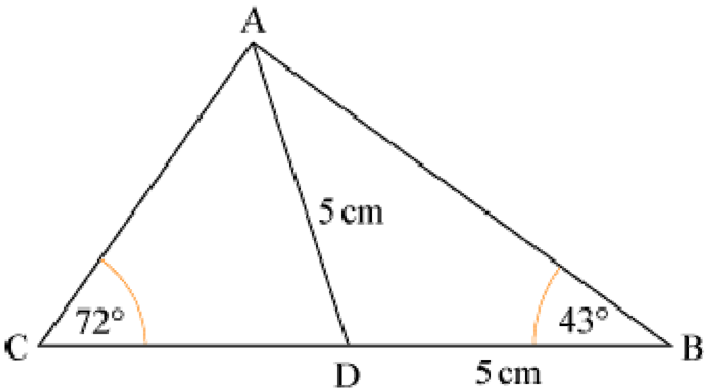
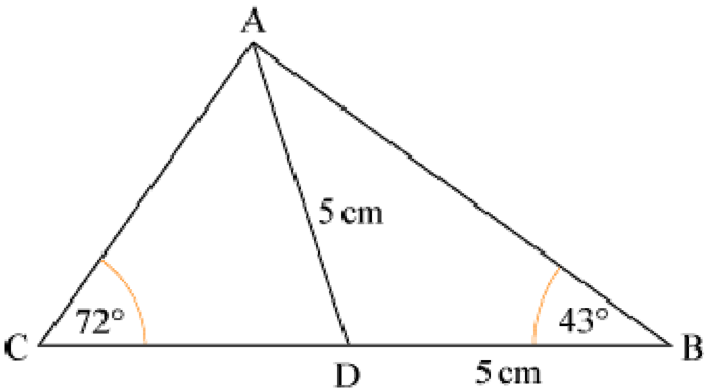
The sine and cosine rule
Exercise A, Question 5

Question:

In the diagram AD = DB = 5 cm, $\angle$ ABC = 43 °
and $\angle$ ACB = 72 ° .
Calculate (a) *AB* and (b) *CD*.



Solution:



(a) In △*ABD*, $\angle$ DAB = 43 ° (isosceles △).
So $\angle$ ADB = 180 ° − (2 × 43 °) = 94 °

As triangle is isosceles you could work with right-angled triangle, but using sine rule $\frac{d}{\sin D} = \frac{a}{\sin A}$

$$\Rightarrow \quad \frac{\text{AB}}{\sin 94°} = \frac{5}{\sin 43°}$$

$$\Rightarrow \quad \text{AB} = \frac{5 \sin 94°}{\sin 43°} = 7.31 \text{ cm (3 s.f.)}$$

(b) In △*ADC*, $\angle$ ADC = 180 ° − 94 ° = 86 ° (angles on a straight line).
So $\angle$ CAD = 180 ° − (72 + 86) ° = 22 °

Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\Rightarrow \quad \frac{\text{CD}}{\sin 22°} = \frac{5}{\sin 72°}$$

$\Rightarrow \quad CD = \frac{5 \sin 22°}{\sin 72°} = 1.97 \text{ cm (3 s.f.)}$

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The sine and cosine rule
Exercise B, Question 1

Question:

(*Note:* Give answers to 3 significant figures, unless they are exact.)

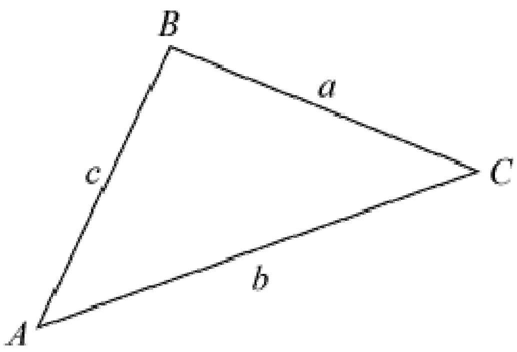
In each of the following sets of data for a triangle *ABC*, find the value of *x*:

(a) AB = 6 cm, BC = 9 cm, $\angle$ BAC = 117 ° , $\angle$ ACB = *x* ° .

(b) AC = 11 cm, BC = 10 cm, $\angle$ ABC = 40 ° , $\angle$ CAB = *x* ° .

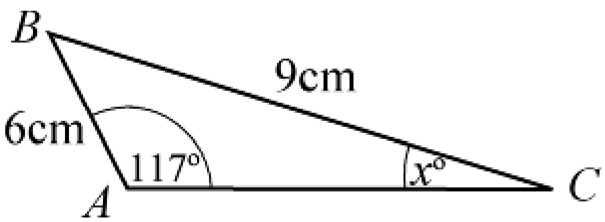
(c) AB = 6 cm, BC = 8 cm, $\angle$ BAC = 60 ° , $\angle$ ACB = *x* ° .

(d) AB = 8.7 cm, AC = 10.8 cm, $\angle$ ABC = 28 ° , $\angle$ BAC = *x* ° .



Solution:

(a)



Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

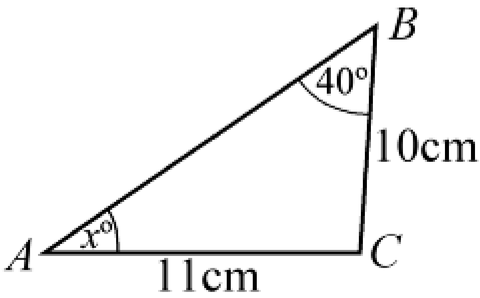
$\Rightarrow \frac{\sin x°}{6} = \frac{\sin 117°}{9}$

$\Rightarrow \sin x° = \frac{6 \sin 117°}{9}$ (= 0.5940...)

$\Rightarrow x° = \sin^{-1}\left(\frac{6 \sin 117°}{9}\right) = 36.4°$ (3 s.f.)

$\Rightarrow x = 36.4$

(b)



Using $\frac{\sin A}{a} = \frac{\sin B}{b}$

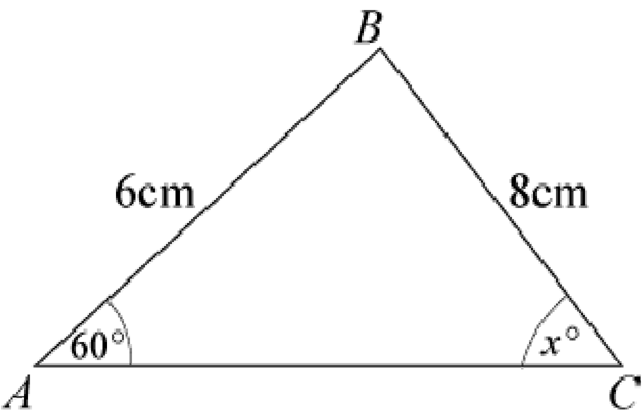
$\Rightarrow \quad \frac{\sin x°}{10} = \frac{\sin 40°}{11}$

$\Rightarrow \quad \sin x° = \frac{10 \sin 40°}{11}$ (= 0.5843...)

$\Rightarrow \quad x° = \sin^{-1}\left(\frac{10 \sin 40°}{11}\right) = 35.8°$ (3 s.f.)

$\Rightarrow \quad x = 35.8$

(c)



Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

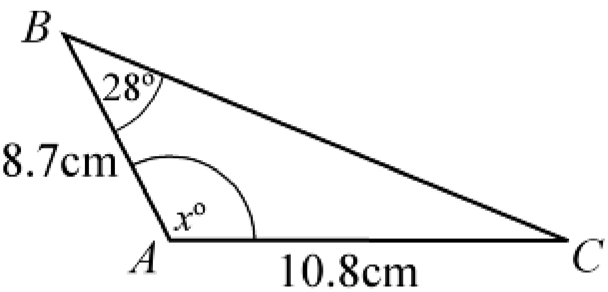
$\Rightarrow \quad \frac{\sin x°}{6} = \frac{\sin 60°}{8}$

$\Rightarrow \quad \sin x° = \frac{6 \sin 60°}{8}$ (= 0.6495...)

$\Rightarrow \quad x° = \sin^{-1}\left(\frac{6 \sin 60°}{8}\right) = 40.5°$ (3 s.f.)

$\Rightarrow \quad x = 40.5$

(d)



Using $\frac{\sin C}{c} = \frac{\sin B}{b}$

$\Rightarrow \quad \frac{\sin C°}{8.7} = \frac{\sin 28°}{10.8}$

$\Rightarrow \quad \sin C° = \frac{8.7 \sin 28°}{10.8} (= 0.3781...)$

$\Rightarrow \quad C° = \sin^{-1} \left(\frac{8.7 \sin 28°}{10.8} \right)$

$\Rightarrow \quad C = 22.2°$ (3 s.f.)

$\Rightarrow \quad x° = 180° - (28 + 22.2)° = 129.8° = 130°$ (3 s.f.)

$\Rightarrow \quad x = 130$

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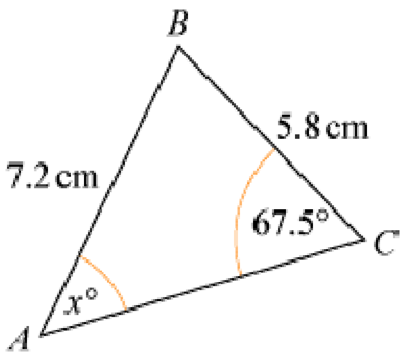
The sine and cosine rule
Exercise B, Question 2

Question:

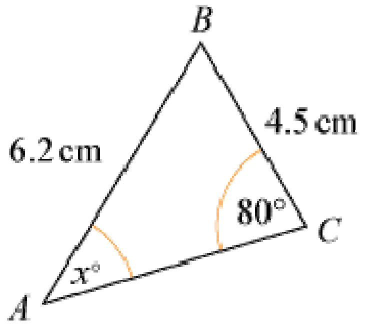
(*Note:* Give answers to 3 significant figures, unless they are exact.)

In each of the diagrams shown below, work out the value of x:

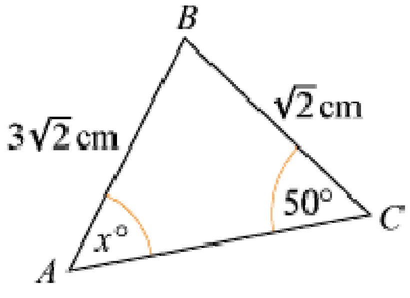
(a)



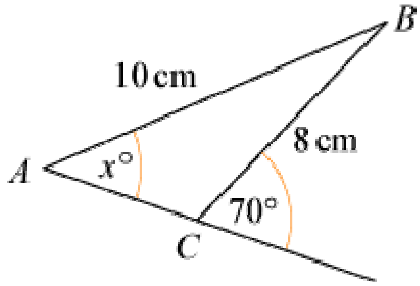
(b)



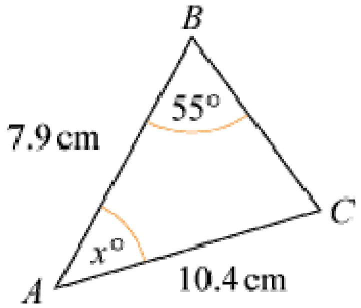
(c)



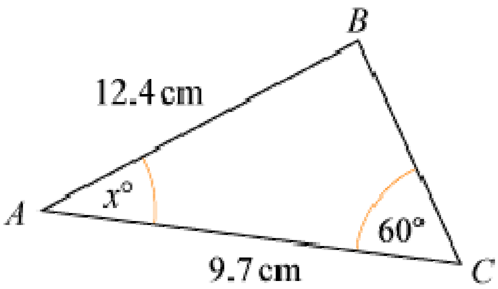
(d)



(e)

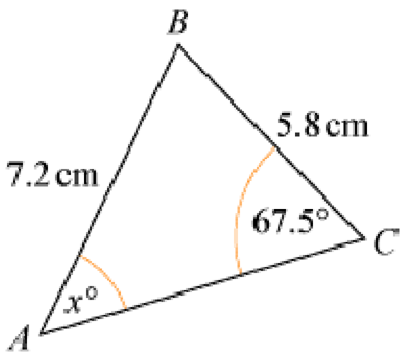


(f)



Solution:

(a)



Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

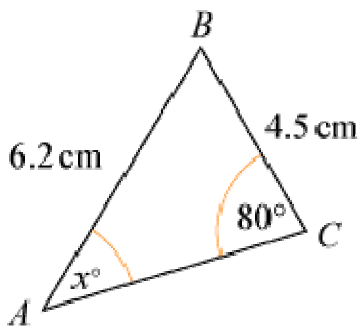
$\Rightarrow \quad \frac{\sin x°}{5.8} = \frac{\sin 67.5°}{7.2}$

$\Rightarrow \quad \sin x° = \frac{5.8 \sin 67.5°}{7.2} (= 0.7442...)$

$\Rightarrow \quad x° = \sin^{-1}\left(\frac{5.8 \sin 67.5°}{7.2}\right) = 48.09°$

$\Rightarrow \quad x = 48.1$ (3 s.f.)

(b)



Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

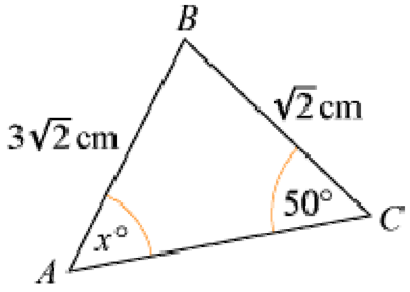
$\Rightarrow \quad \frac{\sin x°}{4.5} = \frac{\sin 80°}{6.2}$

$\Rightarrow \quad \sin x° = \frac{4.5 \sin 80°}{6.2} (= 0.7147...)$

$\Rightarrow \quad x° = \sin^{-1}\left(\frac{4.5 \sin 80°}{6.2}\right) = 45.63°$

$\Rightarrow \quad x = 45.6$ (3 s.f.)

(c)



Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

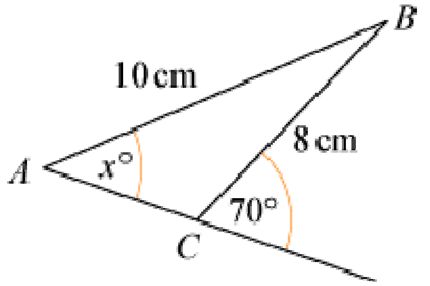
$\Rightarrow \quad \frac{\sin x°}{\sqrt{2}} = \frac{\sin 50°}{3\sqrt{2}}$

$\Rightarrow \quad \sin x° = \frac{\sqrt{2} \sin 50°}{3\sqrt{2}} \quad (= 0.2553 \quad \ldots \quad)$

$\Rightarrow \quad x° = \sin^{-1}\left(\frac{\sin 50°}{3}\right) = 14.79°$

$\Rightarrow \quad x = 14.8$ (3 s.f.)

(d)



Angle ACB = $180° - 70° = 110°$

Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

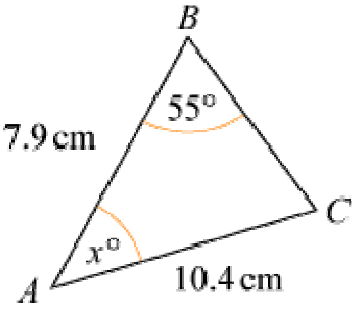
$\Rightarrow \quad \frac{\sin x°}{8} = \frac{\sin 110°}{10}$

$\Rightarrow \quad \sin x° = \frac{8 \sin 110°}{10} (= 0.7517...)$

$\Rightarrow \quad x° = \sin^{-1}\left(\frac{8 \sin 110°}{10}\right) = 48.74°$

$\Rightarrow \quad x = 48.7$ (3 s.f.)

(e)



Using $\frac{\sin C}{c} = \frac{\sin B}{b}$

$\Rightarrow \quad \frac{\sin C}{7.9} = \frac{\sin 55°}{10.4}$

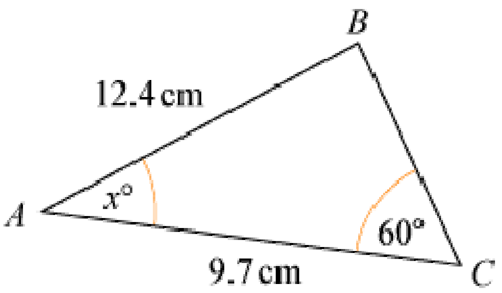
$\Rightarrow \quad \sin C = \frac{7.9 \sin 55°}{10.4} (= 0.6222...)$

$\Rightarrow \quad C = \sin^{-1}\left(\frac{7.9 \sin 55°}{10.4}\right) = 38.48°$

$x° = 180° - (55 + C)°$

$\Rightarrow \quad x = 86.52 = 86.5$ (3 s.f.)

(f)



Using $\frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow \quad \frac{\sin B}{9.7} = \frac{\sin 60°}{12.4}$

$\Rightarrow \quad \sin B = \frac{9.7 \sin 60°}{12.4} (= 0.6774...)$

$\Rightarrow \quad B = 42.65°$

$x° = 180° - (60 + B)° = 77.35°$

$\Rightarrow \quad x = 77.4$ (3 s.f.)

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Edexcel Modular Mathematics for AS and A-Level

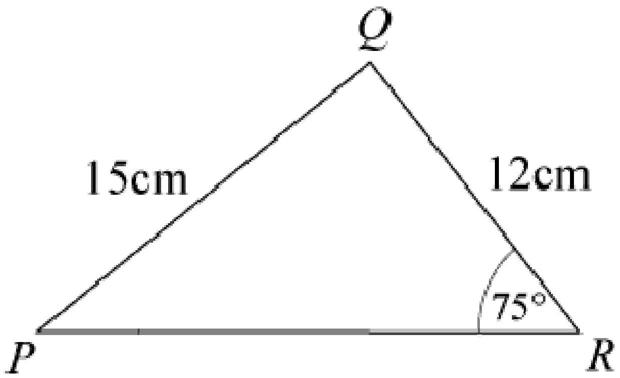
The sine and cosine rule
Exercise B, Question 3

Question:

(*Note*: Give answers to 3 significant figures, unless they are exact.)

In $\triangle PQR$, PQ = 15 cm, QR = 12 cm and $\angle$ PRQ = 75 ° . Find the two remaining angles.

Solution:



Using $\frac{\sin P}{p} = \frac{\sin R}{r}$

$\Rightarrow \quad \frac{\sin P}{12} = \frac{\sin 75°}{15}$

$\Rightarrow \quad \sin P = \frac{12 \sin 75°}{15} (= 0.7727...)$

$\Rightarrow \quad P = \sin^{-1}\left(\frac{12 \sin 75°}{15}\right) = 50.60°$

Angle QPR = 50.6 ° (3 s.f.)
Angle PQR = 180 ° − (75 + 50.6) ° = 54.4 ° (3 s.f)

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Edexcel Modular Mathematics for AS and A-Level

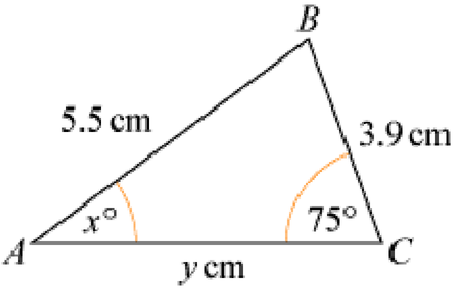
The sine and cosine rule
Exercise B, Question 4

Question:

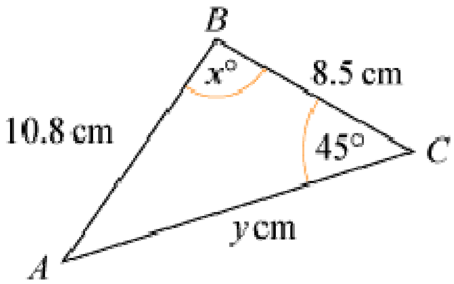
(*Note:* Give answers to 3 significant figures, unless they are exact.)

In each of the following diagrams work out the values of *x* and *y*:

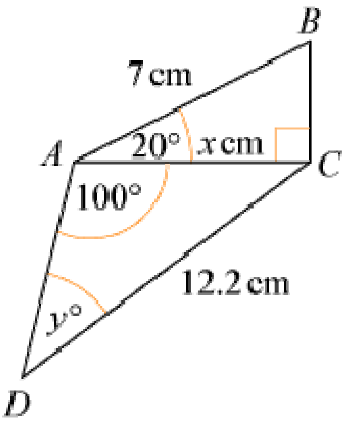
(a)



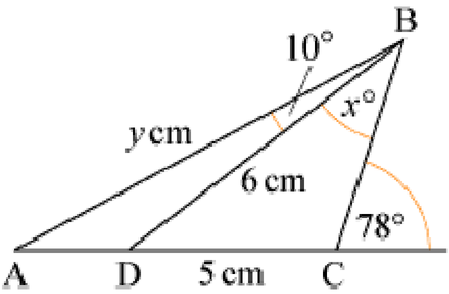
(b)



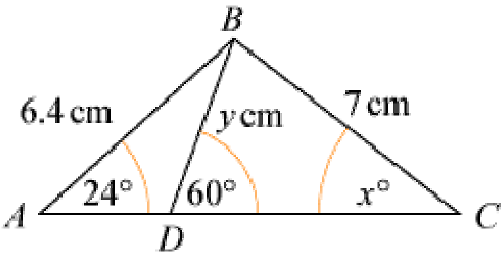
(c)



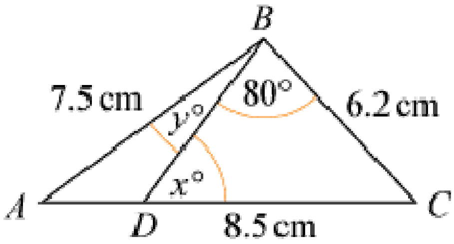
(d)



(e)

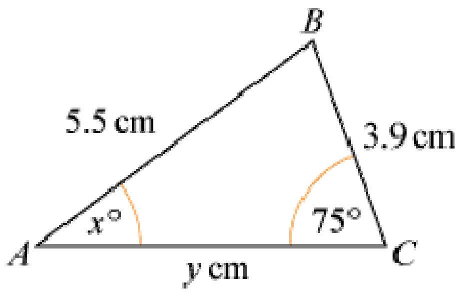


(f)



Solution:

(a)



Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\Rightarrow \quad \frac{\sin x°}{3.9} = \frac{\sin 75°}{5.5}$

$\Rightarrow \quad \sin x° = \frac{3.9 \sin 75°}{5.5}$

$\Rightarrow \quad x° = \sin^{-1}\left(\frac{3.9 \sin 75°}{5.5}\right) = 43.23°$

$\Rightarrow \quad x = 43.2$ (3 s.f.)

So $\angle ABC = 180° - (75 + 43.2)° = 61.8°$

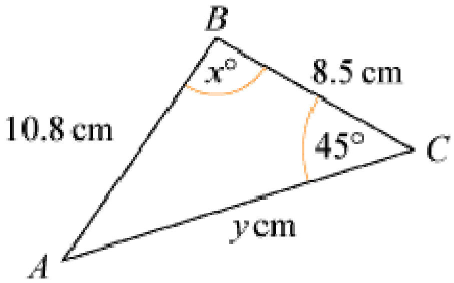
Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \quad \frac{y}{\sin 61.8°} = \frac{5.5}{\sin 75°}$

$\Rightarrow \quad y = \frac{5.5 \sin 61.8°}{\sin 75°} = 5.018$

$\Rightarrow \quad y = 5.02$(3 s.f.)

(b)



Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\Rightarrow \quad \frac{\sin A}{8.5} = \frac{\sin 45°}{10.8}$

$\Rightarrow \quad \sin A = \frac{8.5 \sin 45°}{10.8}$

$\Rightarrow \quad A = \sin^{-1}\left(\frac{8.5 \sin 45°}{10.8}\right) = 33.815°$

$x° = 180° - (45 + A)° = 101.2°$

$\Rightarrow \quad x = 101$ (3 s.f.)

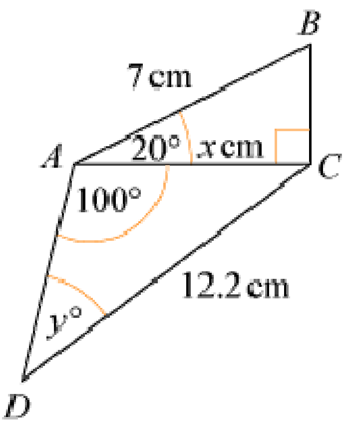
Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \quad \frac{y}{\sin x°} = \frac{10.8}{\sin 45°}$

$\Rightarrow \quad y = \frac{10.8 \sin x°}{\sin 45°} = 14.98$

$\Rightarrow \quad y = 15.0$ (3 s.f.)

(c)



In $\triangle ABC$, $\frac{x}{7} = \cos 20° \quad \Rightarrow \quad x = 7 \cos 20° = 6.578 = 6.58$ (3 s.f.)

In $\triangle ADC$, using $\frac{\sin D}{d} = \frac{\sin A}{a}$

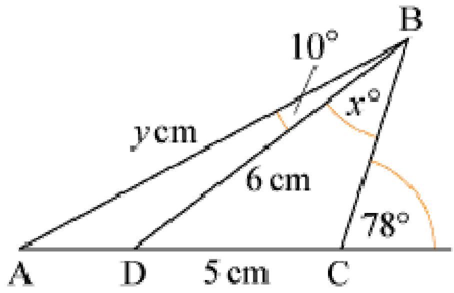
$\Rightarrow \quad \frac{\sin y°}{x} = \frac{\sin 100°}{12.2}$

$\Rightarrow \quad \sin y° = \frac{x \sin 100°}{12.2}$

$\Rightarrow \quad y° = \sin^{-1}\left(\frac{x \sin 100°}{12.2}\right) = 32.07°$

$\Rightarrow \quad y = 32.1$ (3 s.f.)

(d)



In $\triangle BDC$, $\angle C = 180° - 78° = 102°$

Using $\frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow \quad \frac{\sin x°}{5} = \frac{\sin 102°}{6}$

$\Rightarrow \quad \sin x° = \frac{5 \sin 102°}{6}$

$\Rightarrow \quad x° = \sin^{-1}\left(\frac{5 \sin 102°}{6}\right) = 54.599°$

$\Rightarrow \quad x = 54.6$ (3 s.f.)

In $\triangle ABC$, $\angle \text{BAC} = 180° - 102° - (10 + x)° = 13.4°$

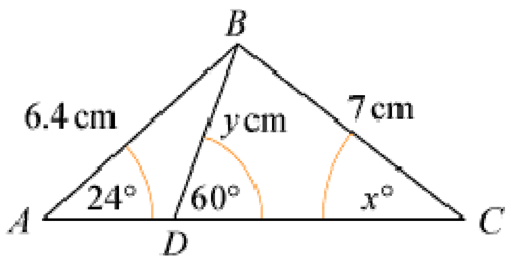
So $\angle \text{ADB} = 180° - 10° - 13.4° = 156.6°$

Using $\frac{d}{\sin D} = \frac{a}{\sin A}$ in $\triangle ABD$

$\Rightarrow \quad \frac{y}{\sin 156.6°} = \frac{6}{\sin 13.4°}$

$\Rightarrow \quad y = \frac{6 \sin 156.6°}{\sin 13.4°} = 10.28 = 10.3$ (3 s.f.)

(e)



In $\triangle ABC$, using $\frac{\sin C}{c} = \frac{\sin A}{a}$

$\Rightarrow \quad \frac{\sin x°}{6.4} = \frac{\sin 24°}{7}$

$\Rightarrow \quad \sin x° = \frac{6.4 \sin 24°}{7}$

$\Rightarrow \quad x° = \sin^{-1}\left(\frac{6.4 \sin 24°}{7}\right) = 21.83°$

$\Rightarrow \quad x = 21.8$ (3 s.f.)

In $\triangle ABD$, using $\frac{a}{\sin A} = \frac{d}{\sin D}$

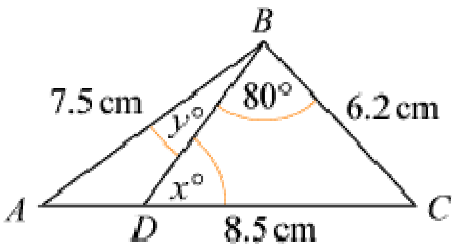
$\Rightarrow \quad \frac{y}{\sin 24°} = \frac{6.4}{\sin 120°}$

$\Rightarrow \quad y = \dfrac{6.4 \sin 24°}{\sin 120°} = 3.0058$

$\Rightarrow \quad y = 3.01$ (3 s.f.)

(The above approach finds the two values independently. You could find *y* first and then use it to find *x*, but then if *y* is wrong then so will *x* be.)

(f)



Using $\dfrac{\sin D}{d} = \dfrac{\sin B}{b}$ in $\triangle BDC$

$\Rightarrow \quad \dfrac{\sin x°}{6.2} = \dfrac{\sin 80°}{8.5}$

$\Rightarrow \quad x° = \dfrac{6.2 \sin 80°}{8.5}$

$\Rightarrow \quad x° = \sin^{-1}\left(\dfrac{6.2 \sin 80°}{8.5}\right) = 45.92°$

$\Rightarrow \quad x = 45.9$ (3 s.f.)

In $\triangle ABC$, $\angle ACB = 180° - (80 + x)° = 54.08°$

Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$\Rightarrow \quad \dfrac{\sin A}{6.2} = \dfrac{\sin 54.08°}{7.5}$

$\Rightarrow \quad \sin A = \dfrac{6.2 \sin 54.08°}{7.5}$

$\Rightarrow \quad A = \sin^{-1}\left(\dfrac{6.2 \sin 54.08°}{7.5}\right) = 42.03°$

So $y° = 180° - (42.03 + 134.1)° = 3.87$ (3 s.f.)

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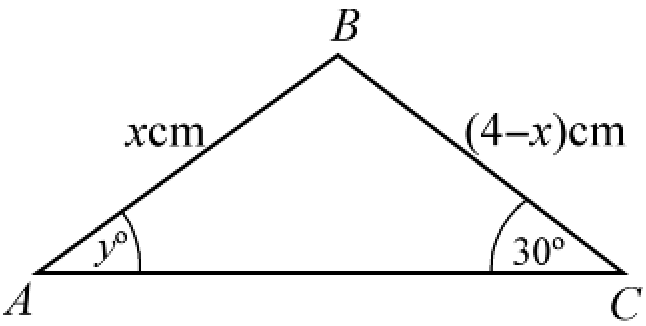
The sine and cosine rule
Exercise B, Question 5

Question:

In $\triangle ABC$, AB = x cm, BC = $(4 - x)$ cm, $\angle$ BAC = $y°$ and $\angle$ BCA = $30°$.

Given that $\sin y° = \frac{1}{\sqrt{2}}$, show that $x = 4(\sqrt{2} - 1)$.

Solution:



Using $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\Rightarrow \frac{4 - x}{\sin y°} = \frac{x}{\sin 30°}$

$\Rightarrow (4 - x) \sin 30° = x \sin y°$

$\Rightarrow (4 - x) \times \frac{1}{2} = x \times \frac{1}{\sqrt{2}}$

Multiply throughout by 2:

$4 - x = x\sqrt{2}$

$x + \sqrt{2}x = 4$

$x(1 + \sqrt{2}) = 4$

$x = \frac{4}{1 + \sqrt{2}}$

Multiply 'top and bottom' by $\sqrt{2} - 1$:

$x = \frac{4(\sqrt{2} - 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)} = \frac{4(\sqrt{2} - 1)}{2 - 1} = 4(\sqrt{2} - 1)$

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The sine and cosine rule
Exercise C, Question 1

Question:

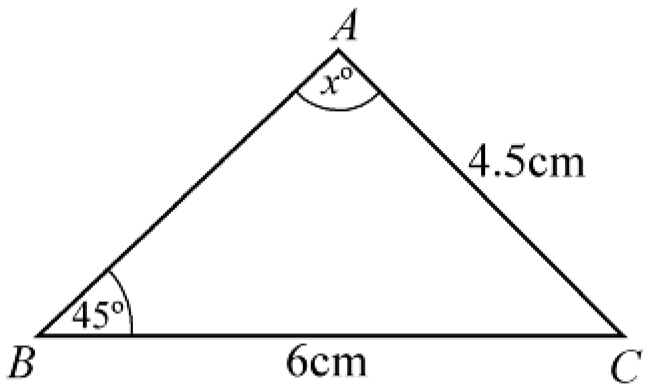
(Give answers to 3 significant figures.)
In $\triangle ABC$, BC = 6 cm, AC = 4.5 cm and $\angle$ ABC = 45 ° :

(a) Calculate the two possible values of $\angle$ BAC.

(b) Draw a diagram to illustrate your answers.

Solution:

(a)



$x > 45°$
So there are two possible results.

Using $\frac{\sin A}{a} = \frac{\sin B}{b}$

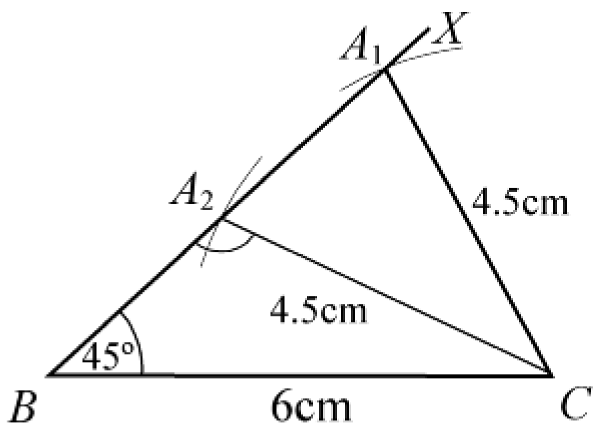
$\frac{\sin x°}{6} = \frac{\sin 45°}{4.5}$

$\sin x° = \frac{6 \sin 45°}{4.5}$

$x° = \sin^{-1}\left(\frac{6 \sin 45°}{4.5}\right)$ or $180° - \sin^{-1}\left(\frac{6 \sin 45°}{4.5}\right)$

$x° = 70.5°$ (3 s.f.) or $109.5°$

(b)



Draw BC = 6 cm.

Measure angle of 45° at *B* (*BX*).
Put compass point at *C* and open out to 4.5 cm. Where arc meets *BX* are the two possible positions of *A*.

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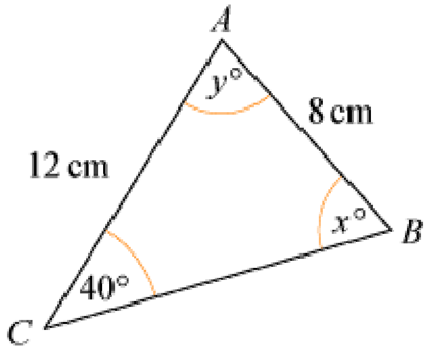
The sine and cosine rule
Exercise C, Question 2

Question:

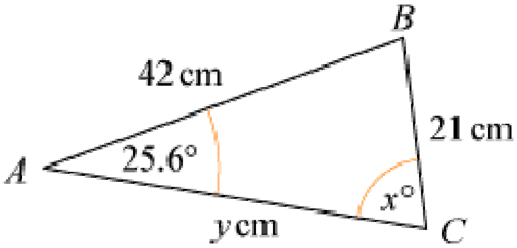
(Give answers to 3 significant figures.)

In each of the diagrams shown below, calculate the possible values of *x* and the corresponding values of *y*:

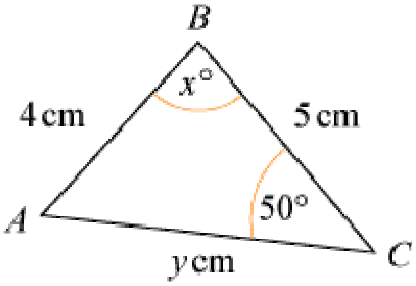
(a)



(b)

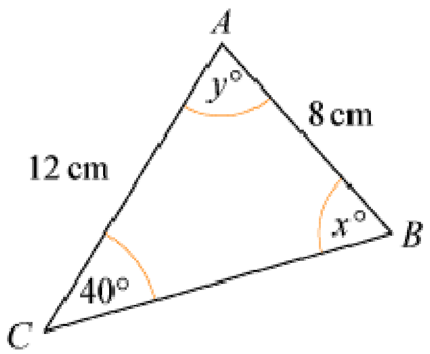


(c)



Solution:

(a)



Using $\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin x°}{12} = \frac{\sin 40°}{8}$

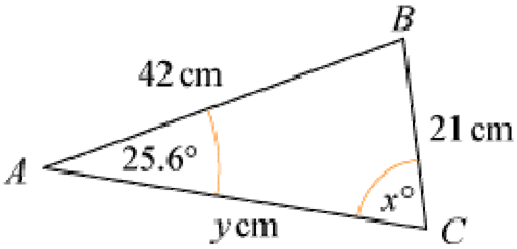
$\sin x° = \frac{12 \sin 40°}{8}$

$x° = \sin^{-1}\left(\frac{12 \sin 40°}{8}\right)$ or $180° - \sin^{-1}\left(\frac{12 \sin 40°}{8}\right)$

$x° = 74.6°$ or $105.4°$
$x = 74.6$ or 105 (3 s.f.)

When $x = 74.6$, $y = 180 - (74.6 + 40) = 180 - 114.6 = 65.4$ (3 s.f.)
When $x = 105.4$, $y = 180 - (105.4 + 40) = 180 - 145.4 = 34.6$ (3 s.f.)

(b)



Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin x°}{42} = \frac{\sin 25.6°}{21}$

$\sin x° = \frac{42 \sin 25.6°}{21}$

$x° = \sin^{-1}(2 \sin 25.6°)$ or $180° - \sin^{-1}(2 \sin 25.6°)$
$x° = 59.79°$ or $120.2°$
$x = 59.8$ or 120 (3 s.f.)

When $x = 59.8$,
angle $B = 180° - (59.8° + 25.6°) = 94.6°$

Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

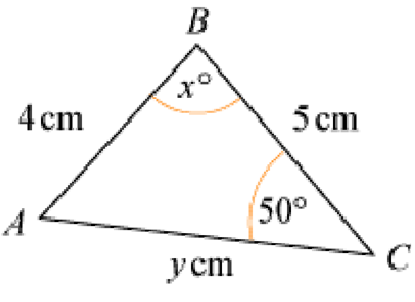
$\frac{y}{\sin 94.6°} = \frac{21}{\sin 25.6°} \Rightarrow y = \frac{21 \sin 94.6°}{\sin 25.6°} = 48.4$ (3 s.f.)

When $x = 120.2$,
angle $B = 180° - (120.2° + 25.6°) = 34.2°$

Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{y}{\sin 34.2°} = \frac{21}{\sin 25.6°} \Rightarrow y = \frac{21 \sin 34.2°}{\sin 25.6°} = 27.3$ (3 s.f.)

(c)



Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin A}{5} = \frac{\sin 50°}{4}$

$\sin A = \frac{5 \sin 50°}{4}$

$A = \sin^{-1}\left(\frac{5 \sin 50°}{4}\right)$ or $180° - \sin^{-1}\left(\frac{5 \sin 50°}{4}\right)$

A = 73.25 or 106.75

When A = 73.247,
$x = 180 - (50 + 73.247) = 56.753 \quad \ldots \quad = 56.8$ (3 s.f.)

Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{y}{\sin x°} = \frac{4}{\sin 50°} \Rightarrow y = \frac{4 \sin x°}{\sin 50°} = 4.37$ (3 s.f.)

When A = 106.75,
$x = 180 - (50 + 106.75) = 23.247 = 23.2$ (3 s.f.)

As above: $y = \frac{4 \sin x°}{\sin 50°} = 2.06$ (3 s.f.)

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The sine and cosine rule
Exercise C, Question 3

Question:

(Give answers to 3 significant figures.)

In each of the following cases $\triangle ABC$ has $\angle ABC = 30°$ and AB = 10 cm:

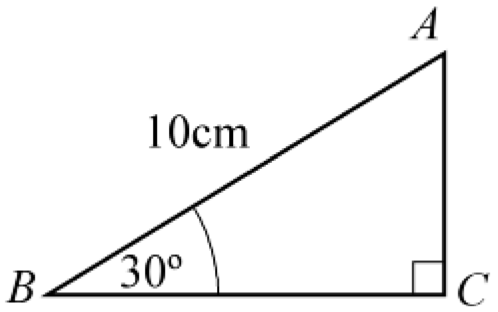
(a) Calculate the least possible length that *AC* could be.

(b) Given that AC = 12 cm, calculate $\angle ACB$.

(c) Given instead that AC = 7 cm, calculate the two possible values of $\angle ACB$.

Solution:

(a)



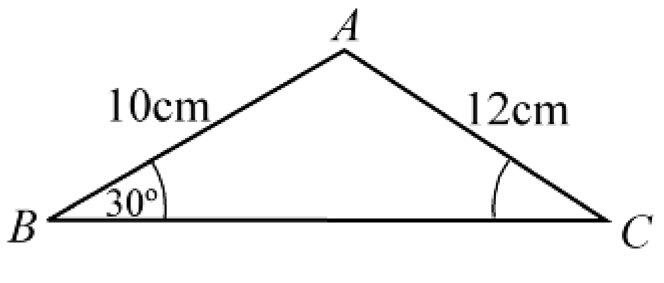
AC is least when it is at right angles to *BC*.

Using $\sin B = \frac{AC}{AB}$

$\sin 30° = \frac{AC}{10}$

$AC = 10 \sin 30° = 5$
AC = 5 cm

(b)



Using $\frac{\sin C}{c} = \frac{\sin B}{b}$

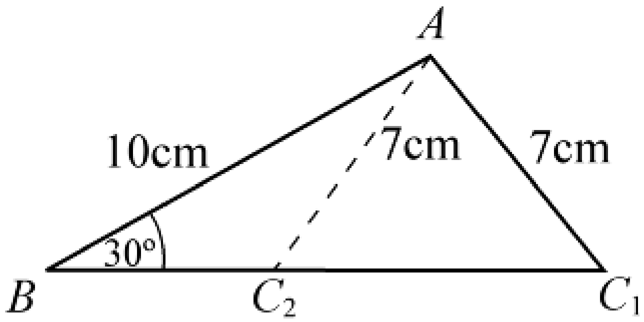
$\frac{\sin C}{10} = \frac{\sin 30°}{12}$

$\sin C = \frac{10 \sin 30°}{12}$

$C = \sin^{-1}\left(\frac{10 \sin 30°}{12}\right) = 24.62°$

$\angle$ ACB = 24.6 ° (3 s.f.)

(c)



As 7 cm < 10 cm, $\angle$ ACB > 30 ° and there are two possible results.
Using 7 cm instead of 12 cm in (b):

$\sin C = \frac{10 \sin 30°}{7}$

$C = \sin^{-1}\left(\frac{10 \sin 30°}{7}\right)$ or $180° - \sin^{-1}\left(\frac{10 \sin 30°}{7}\right)$

$C = 45.58°$ or $134.4°$

$\angle$ ACB = 45.6 ° (3 s.f.) or 134 ° (3 s.f.)

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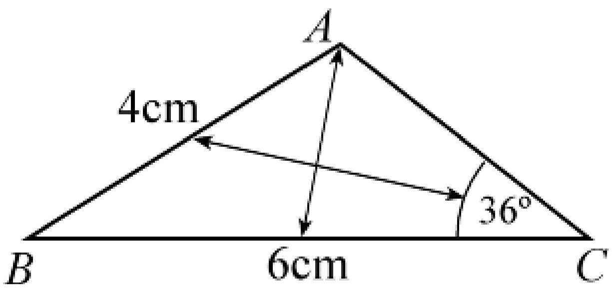
The sine and cosine rule
Exercise C, Question 4

Question:

(Give answers to 3 significant figures.)

Triangle *ABC* is such that AB = 4 cm, BC = 6 cm and $\angle$ ACB = 36 ° . Show that one of the possible values of $\angle$ ABC is 25.8 ° (to 3 s.f.). Using this value, calculate the length of *AC*.

Solution:



As 4 < 6, 36 ° < $\angle$ BAC, so there are two possible values for angle *A*.

Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

$$\frac{\sin A}{6} = \frac{\sin 36°}{4}$$

$$\sin A = \frac{6 \sin 36°}{4}$$

$$A = \sin^{-1}\left(\frac{6 \sin 36°}{4}\right) \text{ or } 180° - \sin^{-1}\left(\frac{6 \sin 36°}{4}\right)$$

$A = 61.845 \ldots°$ or $118.154 \ldots°$

When $A = 118.154 \ldots°$

$\angle ABC = 180° - (36° + 118.154 \ldots°) = 25.846 \ldots° = 25.8°$ (3 s.f.)

Using this value for $\angle$ ABC and $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\frac{AC}{\sin 25.8°} = \frac{4}{\sin 36°}$$

$$\Rightarrow \quad AC = \frac{4 \sin 25.8°}{\sin 36°} = 2.96 \text{ cm (3 s.f.)}$$

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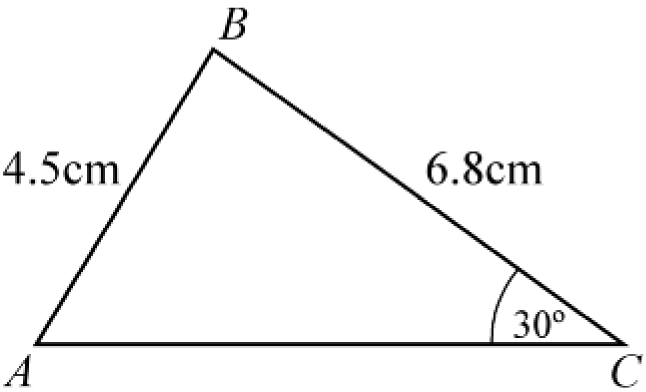
The sine and cosine rule
Exercise C, Question 5

Question:

(Give answers to 3 significant figures.)

Two triangles *ABC* are such that AB = 4.5 cm, BC = 6.8 cm and $\angle$ ACB = 30 ° . Work out the value of the largest angle in each of the triangles.

Solution:



As 6.8 > 4.5, angle *A* > 30 ° and so there are two possible values for *A*.

Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin A}{6.8} = \frac{\sin 30°}{4.5}$

$A = \sin^{-1}\left(\frac{6.8 \sin 30°}{4.5}\right)$ or $180° - \sin^{-1}\left(\frac{6.8 \sin 30°}{4.5}\right)$

$A = 49.07 \ldots °$ or $130.926 \ldots °$

When $A = 49.07 \ldots °$, angle *B* is the largest angle

$\angle ABC = 180° - (30° + 49.07 \ldots °) = 100.9 \ldots ° = 101°$ (3 s.f.)

When $A = 130.926 \ldots °$, this will be the largest angle

$\angle BAC = 131°$ (3 s.f.)

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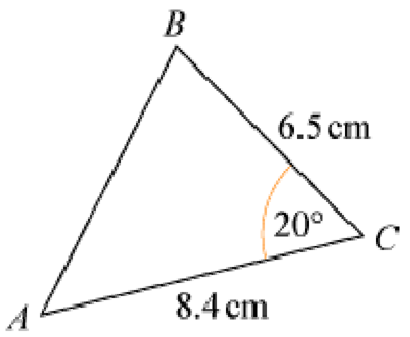
The sine and cosine rule
Exercise D, Question 1

Question:

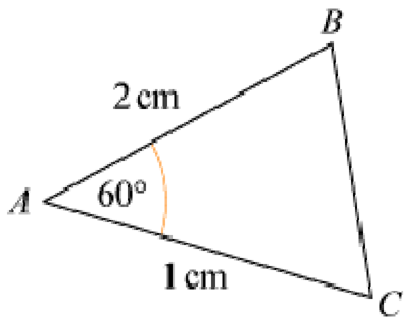
(*Note:* Give answers to 3 significant figures, where appropriate.)

In each of the following triangles calculate the length of the third side:

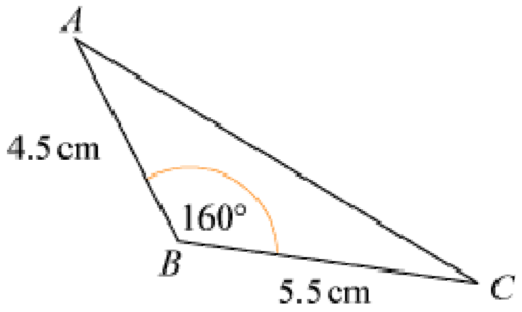
(a)



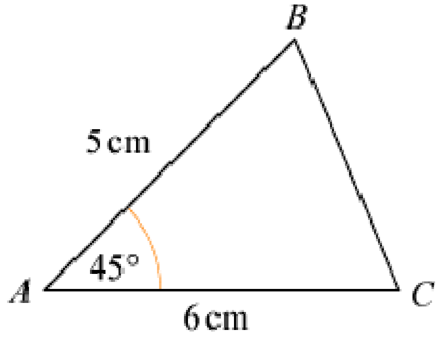
(b)



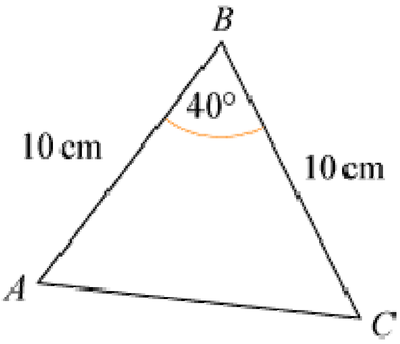
(c)



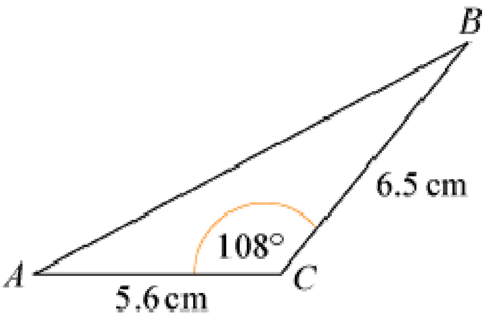
(d)



(e)

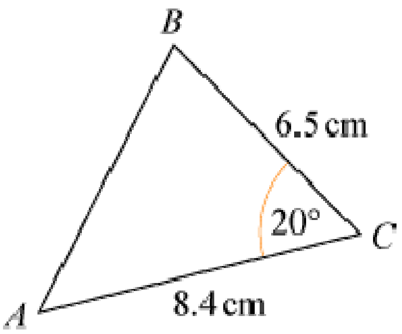


(f)



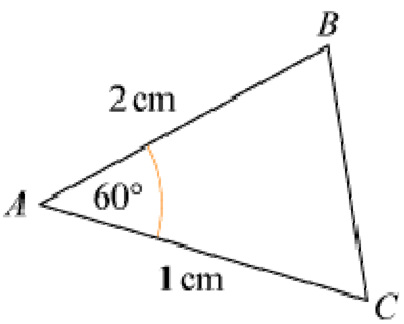
Solution:

(a)



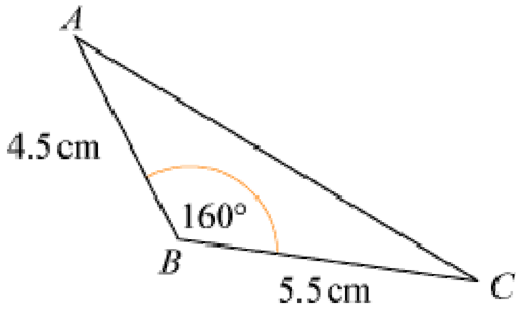
Using $c^2 = a^2 + b^2 - 2ab \cos C$
$AB^2 = 6.5^2 + 8.4^2 - 2 \times 6.5 \times 8.4 \times \cos 20°$
$AB^2 = 10.1955 \ldots$
$AB = \sqrt{10.1955 \ldots} = 3.19$ cm (3 s.f.)

(b)



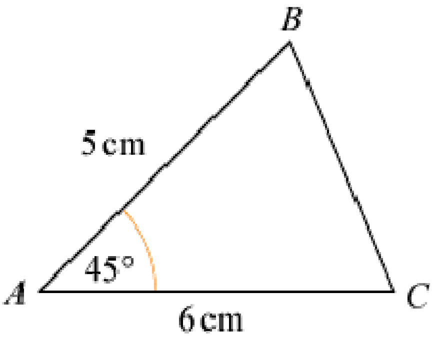
Using $a^2 = b^2 + c^2 - 2bc \cos A$
$BC^2 = 1^2 + 2^2 - 2 \times 1 \times 2 \times \cos 60°$
$BC^2 = 3$
$BC = \sqrt{3} = 1.73$ cm (3 s.f.)

(c)



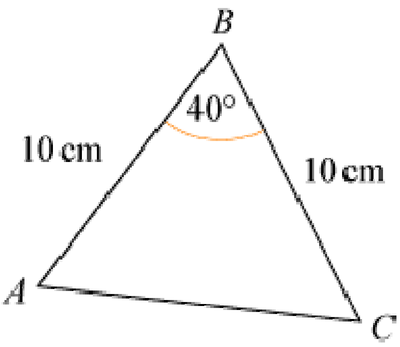
Using $b^2 = a^2 + c^2 - 2ac\ \cos\ B$
$AC^2 = 5.5^2 + 4.5^2 - 2 \times 5.5 \times 4.5 \times \cos\ 160°$
$AC^2 = 97.014\ \ldots$
$AC = \sqrt{97.014\ \ldots} = 9.85$ cm (3 s.f.)

(d)



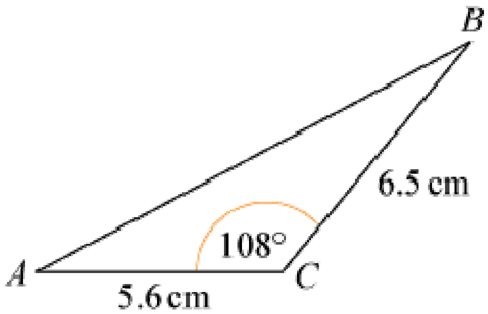
Using $a^2 = b^2 + c^2 - 2bc\ \cos\ A$
$BC^2 = 6^2 + 5^2 - 2 \times 6 \times 5 \times \cos\ 45° = 18.573\ \ldots$
$BC = \sqrt{18.573\ \ldots} = 4.31$ cm (3 s.f.)

(e)



(This is an isosceles triangle and so you could use right-angled triangle work.)
Using $b^2 = a^2 + c^2 - 2ac\ \cos\ B$
$AC^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos\ 40° = 46.791\ \ldots$
$AC = \sqrt{46.791\ \ldots} = 6.84$ cm (3 s.f.)

(f)



Using $c^2 = a^2 + b^2 - 2ab \cos C$
$AB^2 = 6.5^2 + 5.6^2 - 2 \times 6.5 \times 5.6 \times \cos 108° = 96.106 \ldots$
$AB = \sqrt{96.106 \ldots} = 9.80$ cm (3 s.f.)

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Edexcel Modular Mathematics for AS and A-Level

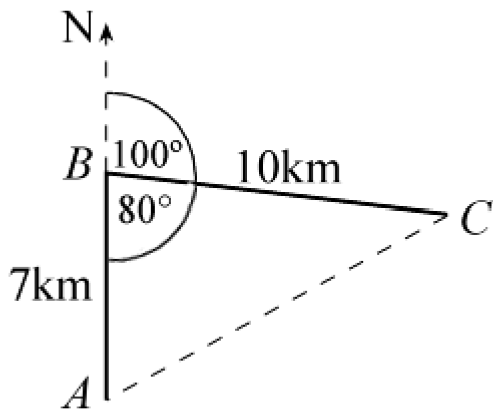
The sine and cosine rule
Exercise D, Question 2

Question:

(*Note:* Give answers to 3 significant figures, where appropriate.)

From a point *A* a boat sails due north for 7 km to *B*. The boat leaves *B* and moves on a bearing of 100° for 10 km until it reaches *C*. Calculate the distance of *C* from *A*.

Solution:



Using the cosine rule: $b^2 = a^2 + c^2 - 2ac\cos B$

$AC^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 80° = 124.689\ldots$

$AC = \sqrt{124.689\ldots} = 11.2$ km (3 s.f.)

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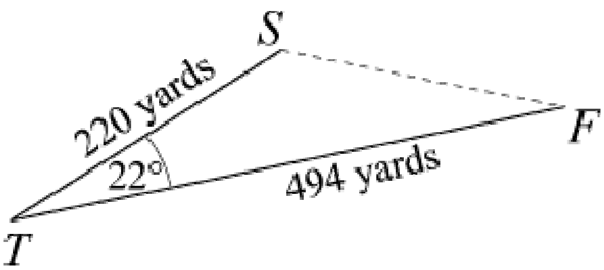
The sine and cosine rule
Exercise D, Question 3

Question:

(*Note:* Give answers to 3 significant figures, where appropriate.)

The distance from the tee, *T*, to the flag, *F*, on a particular hole on a golf course is 494 yards. A golfer's tee shot travels 220 yards and lands at the point *S*, where $\angle STF = 22°$. Calculate how far the ball is from the flag.

Solution:



Using the cosine rule:
$t^2 = f^2 + s^2 - 2fs \cos T$
$SF^2 = 220^2 + 494^2 - 2 \times 220 \times 494 \cos 22° = 90903.317 \ldots$
$SF = \sqrt{90903.317 \ldots} = 301.5 \ldots$ yards $= 302$ yards (3 s.f.)

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

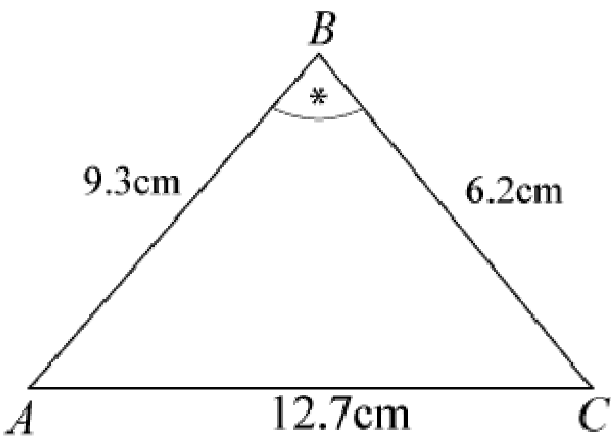
The sine and cosine rule
Exercise D, Question 4

Question:

(*Note:* Give answers to 3 significant figures, where appropriate.)

In $\triangle ABC$, AB = $(x-3)$ cm, BC = $(x+3)$ cm, AC = 8 cm and $\angle$ BAC = 60 ° . Use the cosine rule to find the value of x.

Solution:



Using $a^2 = b^2 + c^2 - 2bc\ \cos\ A$

$(x+3)^2 = (x-3)^2 + 8^2 - 2 \times 8 \times (x-3) \cos\ 60°$

$x^2 + 6x + 9 = x^2 - 6x + 9 + 64 - 8(x-3)$

$x^2 + 6x + 9 = x^2 - 6x + 9 + 64 - 8x + 24$

$6x + 6x + 8x = 64 + 24$

$20x = 88$

$x = \frac{88}{20} = 4.4$

Solutionbank C2
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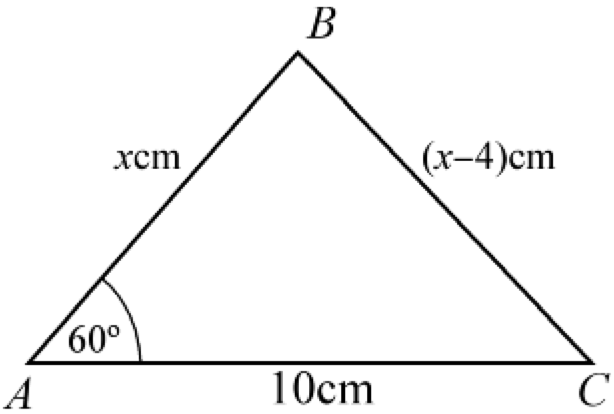
The sine and cosine rule
Exercise D, Question 5

Question:

(*Note:* Give answers to 3 significant figures, where appropriate.)

In $\triangle ABC$, AB = x cm, BC = $(x - 4)$ cm, AC = 10 cm and $\angle$ BAC = 60°. Calculate the value of x.

Solution:



Using $a^2 = b^2 + c^2 - 2bc \cos A$

$(x - 4)^2 = 10^2 + x^2 - 2 \times 10 \times x \cos 60°$

$x^2 - 8x + 16 = 100 + x^2 - 10x$

$10x - 8x = 100 - 16$

$2x = 84$

$x = 42$

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The sine and cosine rule
Exercise D, Question 6

Question:

(*Note:* Give answers to 3 significant figures, where appropriate.)

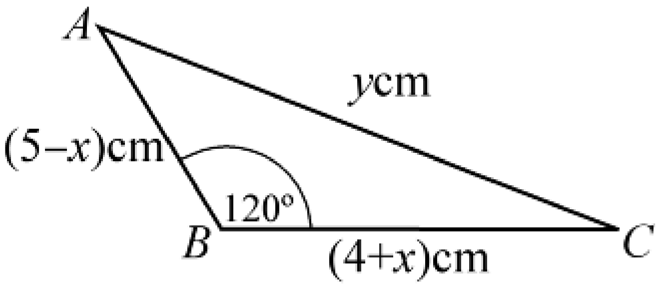
In $\triangle ABC$, AB = $(5-x)$ cm, BC = $(4+x)$ cm, $\angle$ ABC = 120° and AC = y cm.

(a) Show that $y^2 = x^2 - x + 61$.

(b) Use the method of completing the square to find the minimum value of y^2, and give the value of x for which this occurs.

Solution:

(a)



Using $b^2 = a^2 + c^2 - 2ac \cos B$

$y^2 = (4+x)^2 + (5-x)^2 - 2(4+x)(5-x)\cos 120°$

$y^2 = 16 + 8x + x^2 + 25 - 10x + x^2 + (4+x)(5-x)$ (Note: $2 \cos 120° = -1$)

$y^2 = 16 + 8x + x^2 + 25 - 10x + x^2 + 20 + x - x^2 = x^2 - x + 61$

(b) Completing the square: $y^2 = \left(x - \frac{1}{2}\right)^2 + 61 - \frac{1}{4}$

$\Rightarrow \quad y^2 = \left(x - \frac{1}{2}\right)^2 + 60\frac{3}{4}$

Minimum value of y^2 occurs when $\left(x - \frac{1}{2}\right)^2 = 0$, i.e. when $x = \frac{1}{2}$.

So minimum value of $y^2 = 60.75$.

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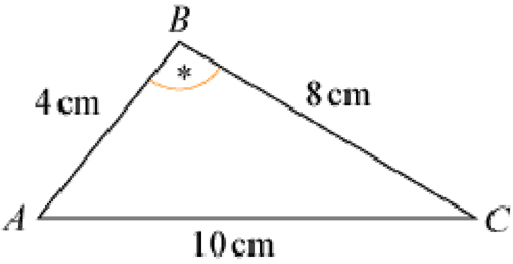
The sine and cosine rule
Exercise E, Question 1

Question:

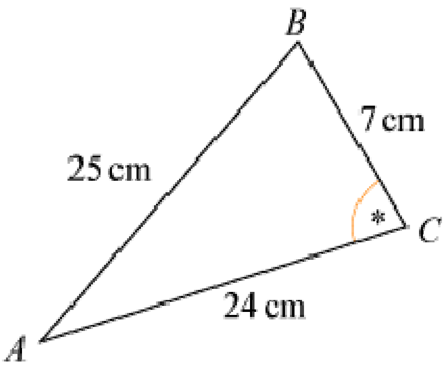
(Give answers to 3 significant figures.)

In the following triangles calculate the size of the angle marked *:

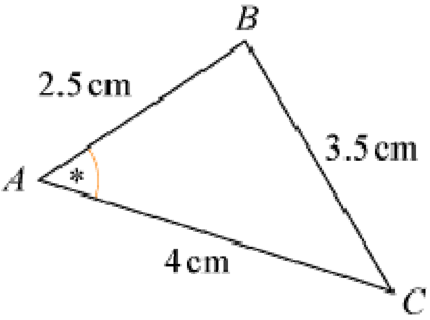
(a)



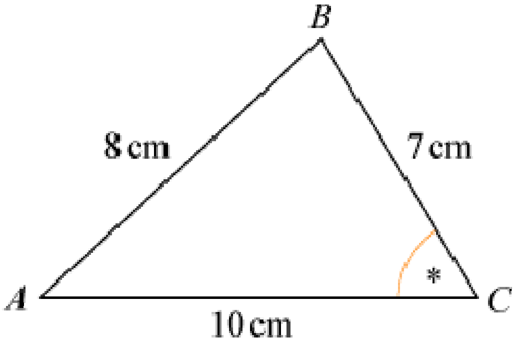
(b)



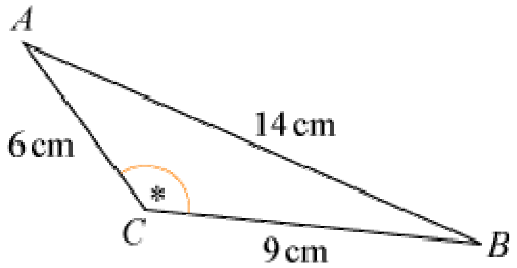
(c)



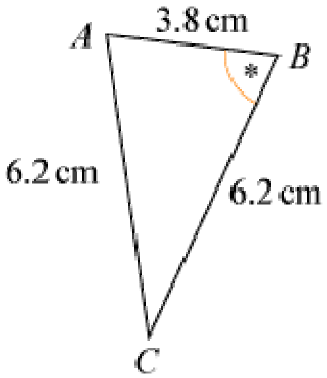
(d)



(e)

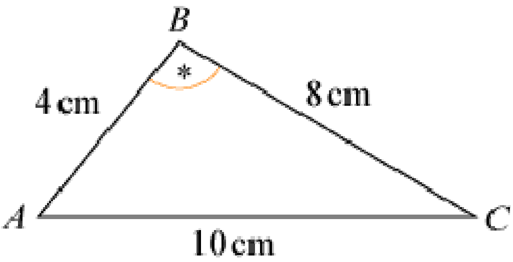


(f)



Solution:

(a)

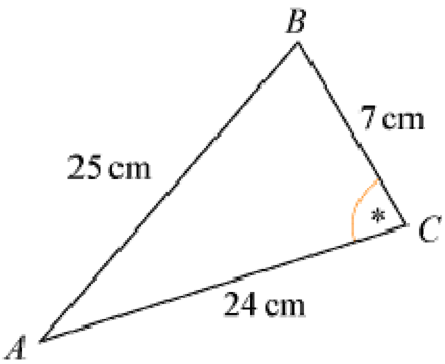


Using $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \frac{8^2 + 4^2 - 10^2}{2 \times 8 \times 4} = -\frac{20}{64} = -\frac{5}{16}$

$B = \cos^{-1}\left(-\frac{5}{16}\right) = 108.2 \ldots ° = 108° \text{ (3 s.f.)}$

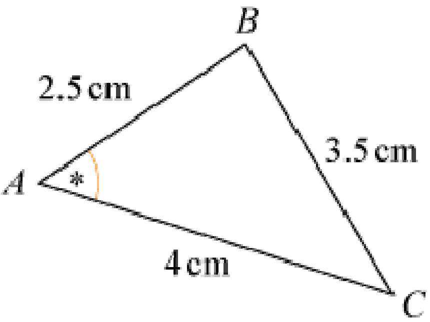
(b)



Using $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos\ C = \frac{7^2 + 24^2 - 25^2}{2 \times 7 \times 24} = 0 \quad \Rightarrow \quad C = 90°$

(c)

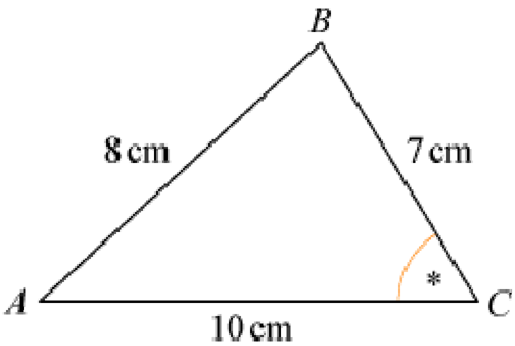


Using $\cos\ A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos\ A = \frac{4^2 + 2.5^2 - 3.5^2}{2 \times 4 \times 2.5} = \frac{1}{2}$

$A = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

(d)

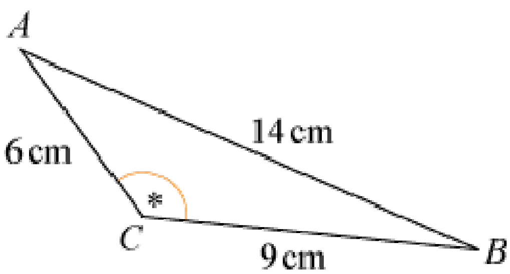


Using $\cos\ C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos\ C = \frac{7^2 + 10^2 - 8^2}{2 \times 7 \times 10} = 0.6071 \quad \ldots$

$C = \cos^{-1}(0.6071...) = 52.6°$ (3 s.f.)

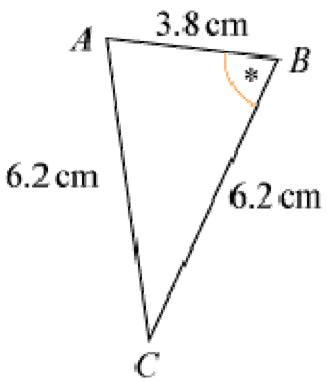
(e)



Using $\cos\ C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos\ C = \frac{9^2 + 6^2 - 14^2}{2 \times 9 \times 6} = -0.7314 \quad \ldots \quad \Rightarrow \quad C = 137°$ (3 s.f.)

(f)



Using $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \frac{6.2^2 + 3.8^2 - 6.2^2}{2 \times 6.2 \times 3.8} = \frac{3.8}{2 \times 6.2} = 0.3064 \ldots \quad \Rightarrow \quad B = 72.2° \text{ (3 s.f.)}$

(This is an isosceles triangle so you could use right-angled triangle trigonometry.)

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

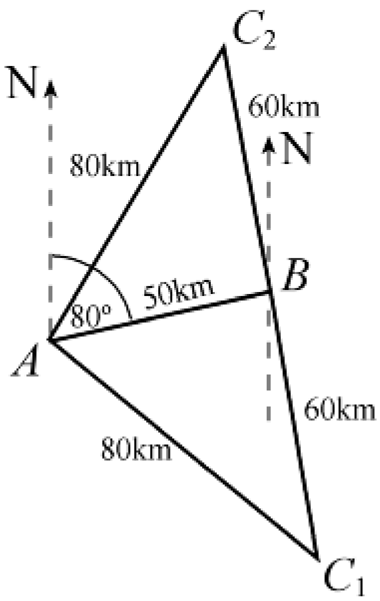
The sine and cosine rule
Exercise E, Question 2

Question:

(Give answers to 3 significant figures.)

A helicopter flies on a bearing of 080 ° from *A* to *B*, where AB = 50 km.
It then flies for 60 km to a point *C*.
Given that *C* is 80 km from *A*, calculate the bearing of *C* from *A*.

Solution:



The bearing of *C* from *B* is not given so there are two possibilities for *C* using the data.
The angle *A* will be the same in each △*ABC*.

Using $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \frac{80^2 + 50^2 - 60^2}{2 \times 80 \times 50} = 0.6625$

$A = 48.5°$
Bearing of *C* from *A* is $80° \pm 48.5° = 128.5°$ or $31.5°$

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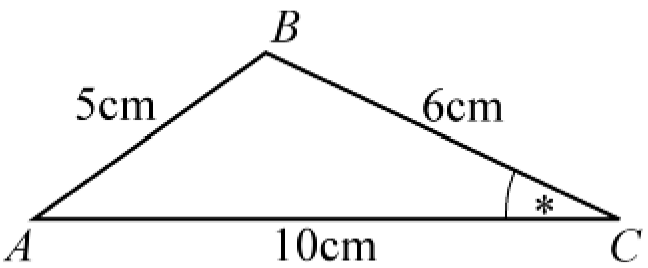
The sine and cosine rule
Exercise E, Question 3

Question:

(Give answers to 3 significant figures.)

In △*ABC*, AB = 5 cm, BC = 6 cm and AC = 10 cm.
Calculate the value of the smallest angle.

Solution:



The smallest angle is *C* as this is opposite *AB*.

Using $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \frac{6^2 + 10^2 - 5^2}{2 \times 6 \times 10} = 0.925$

$C = 22.3°$ (3 s.f.)

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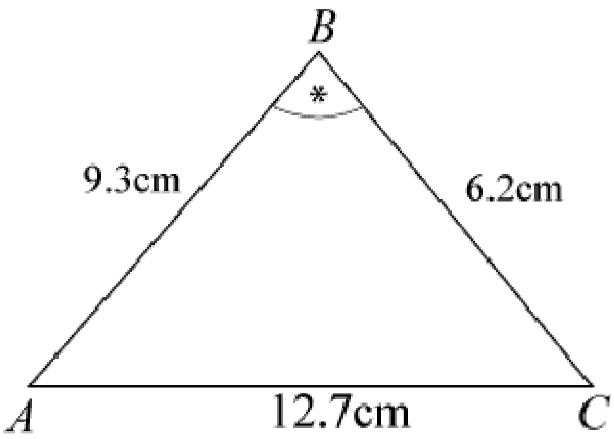
The sine and cosine rule
Exercise E, Question 4

Question:

(Give answers to 3 significant figures.)

In $\triangle ABC$, AB = 9.3 cm, BC = 6.2 cm and AC = 12.7 cm.
Calculate the value of the largest angle.

Solution:



The largest angle is B as it is opposite AC.

Using $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \frac{6.2^2 + 9.3^2 - 12.7^2}{2 \times 6.2 \times 9.3} = -0.3152 \ldots$

$B = 108.37 \ldots = 108°$ (3 s.f.)

Solutionbank C2
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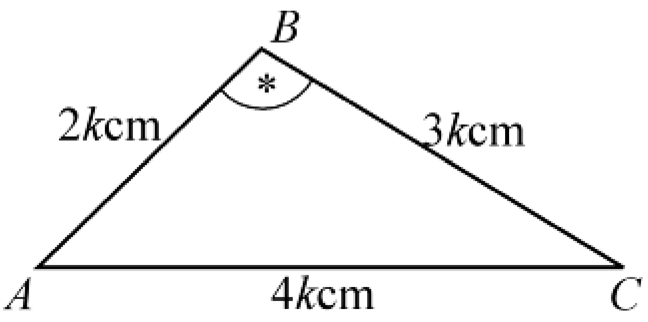
The sine and cosine rule
Exercise E, Question 5

Question:

(Give answers to 3 significant figures.)

The lengths of the sides of a triangle are in the ratio 2:3:4.
Calculate the value of the largest angle.

Solution:



The largest angle will be opposite the side 4*k* cm.

Using $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \frac{9k^2 + 4k^2 - 16k^2}{2 \times 3k \times 2k} = -0.25$

$B = 104.477 \ldots° = 104°$ (3 s.f.)

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The sine and cosine rule
Exercise E, Question 6

Question:

(Give answers to 3 significant figures.)

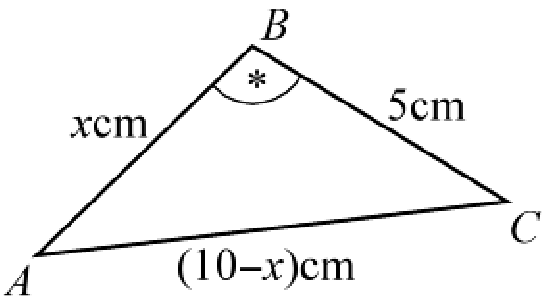
In $\triangle ABC$, AB = x cm, BC = 5 cm and AC = $(10 - x)$ cm:

(a) Show that $\cos \angle ABC = \frac{4x - 15}{2x}$.

(b) Given that $\cos \angle ABC = - \frac{1}{7}$, work out the value of x.

Solution:

(a)



$\cos B = \frac{5^2 + x^2 - (10 - x)^2}{2 \times 5 \times x}$

$= \frac{25 + x^2 - (100 - 20x + x^2)}{10x}$

$= \frac{25 + x^2 - 100 + 20x - x^2}{10x}$

$= \frac{20x - 75}{10x}$

$= \frac{5 (4x - 15)}{10x}$

$= \frac{4x - 15}{2x}$

(b) As $\cos B = - \frac{1}{7}$

$\frac{4x - 15}{2x} = - \frac{1}{7}$

$7 (4x - 15) = - 2x$

$28x - 105 = - 2x$

$30x = 105$

$x = \frac{105}{30} = 3 \frac{1}{2}$

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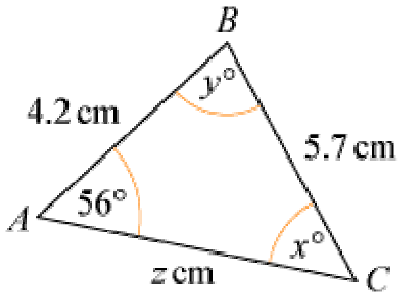
The sine and cosine rule
Exercise F, Question 1

Question:

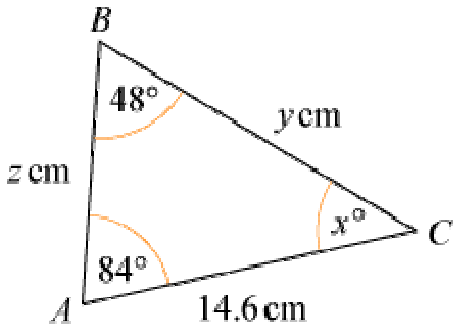
(*Note:* Try to use the neatest method, and give answers to 3 significant figures.)

In each triangle below find the values of x, y and z.

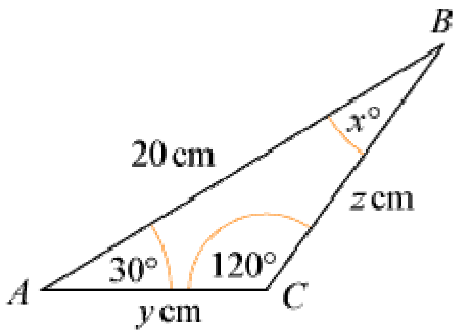
(a)



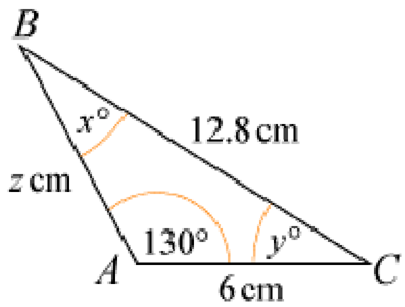
(b)



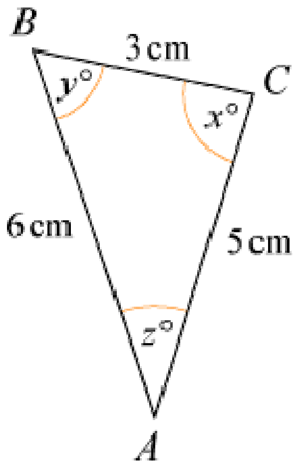
(c)



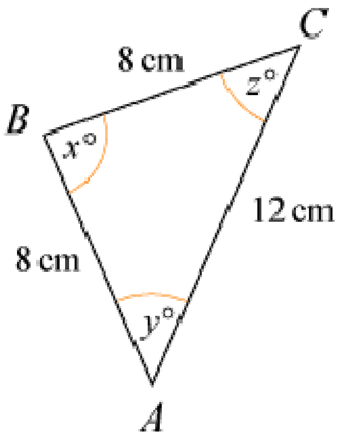
(d)



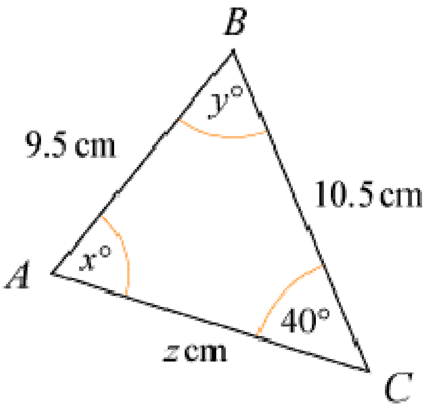
(e)


B
3 cm
C
y°
x°
6 cm
5 cm
z°
A

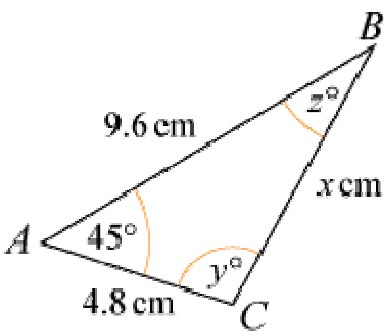
(f)


C
8 cm
z°
B
x°
12 cm
8 cm
y°
A

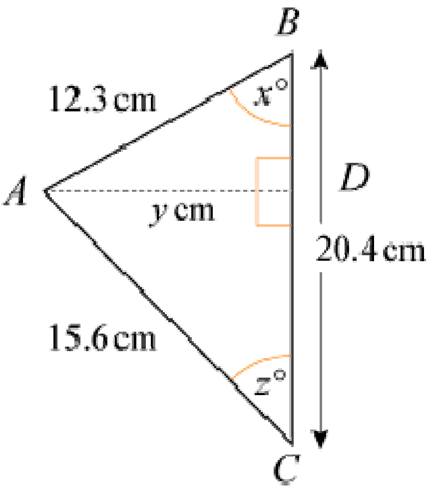
(g)


B
y°
9.5 cm
10.5 cm
A
x°
40°
z cm
C

(h)

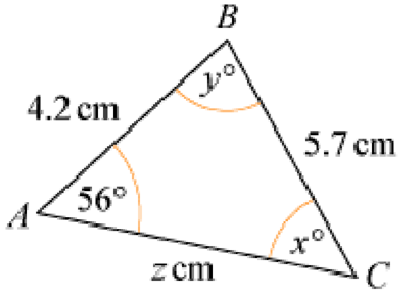

B
z°
9.6 cm
x cm
A
45°
y°
4.8 cm
C

(i)



Solution:

(a)



Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin x°}{4.2} = \frac{\sin 56°}{5.7}$

$\sin x° = \frac{4.2 \sin 56°}{5.7}$

$x° = \sin^{-1}\left(\frac{4.2 \sin 56°}{5.7}\right) = 37.65 \ldots °$

$x = 37.7$ (3 s.f.)

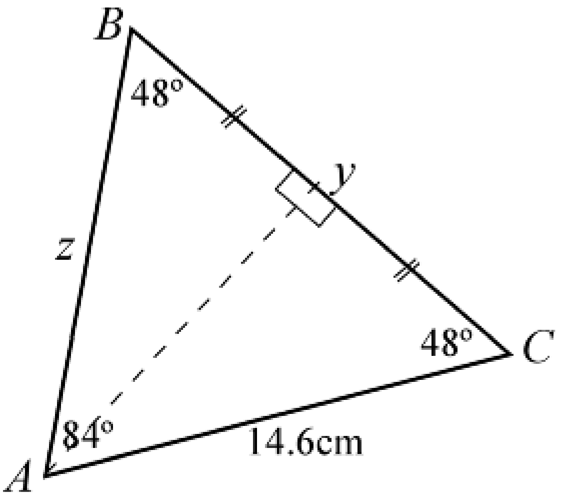
So $y° = 180° - (56° + 37.7°) = 86.3°$

$y = 86.3$ (3 s.f.)

Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{z}{\sin y°} = \frac{5.7}{\sin 56°} \Rightarrow z = \frac{5.7 \sin y°}{\sin 56°} = 6.86$ (3 s.f.)

(b) $x° = 180° - (48° + 84°) = 48° \Rightarrow x = 48$

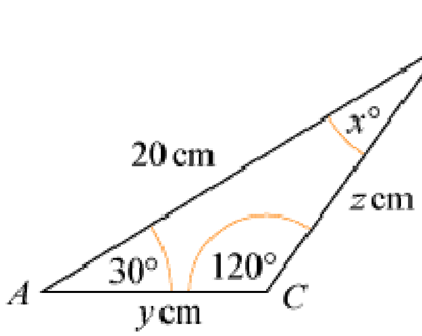


As $\angle B = \angle c$, $z = 14.6$
Using the line of symmetry through A

$$\cos 48° = \frac{\frac{y}{2}}{14.6}$$

$\Rightarrow \quad y = 29.2 \cos 48° = 19.5$ (3 s.f.)

(c)

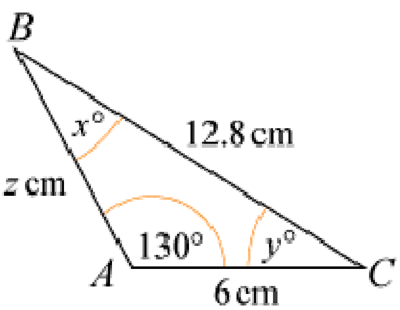


$x° = 180° - (120° + 30°) = 30°$
Using the line of symmetry through C

$\cos 30° = \frac{10}{y} \quad \Rightarrow \quad y = \frac{10}{\cos 30°} = 11.5$ (3 s.f.)

As $\triangle ABC$ is isosceles with AC = CB, $z = 11.5$ (3 s.f.)

(d)



Using $\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 130°}{12.8} = \frac{\sin x°}{6} \quad \Rightarrow \quad \sin x° = \frac{6 \sin 130°}{12.8} = 0.35908 \quad \ldots$

$\Rightarrow \quad x = 21.0$ (3 s.f.)

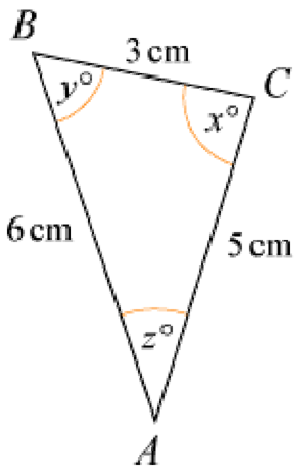
So $y° = 180° - (130° + x°) = 28.956\ldots°$ $\Rightarrow$ $y = 29.0$ (3 s.f.)

Using $\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{z}{\sin y°} = \frac{12.8}{\sin 130°}$

$\Rightarrow$ $z = \frac{12.8 \sin y°}{\sin 130°} = 8.09$ (3 s.f.)

(e)



Using $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos x° = \frac{3^2 + 5^2 - 6^2}{2 \times 3 \times 5} = -0.06$

$x = 93.8$ (3 s.f.)

Using $\frac{\sin B}{b} = \frac{\sin C}{c}$

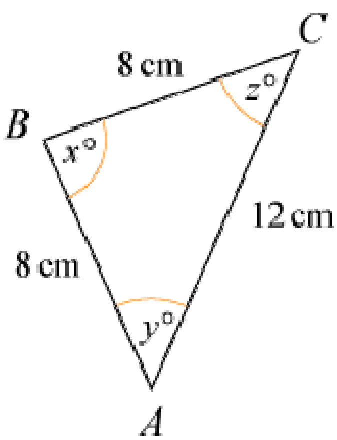
$\frac{\sin y°}{5} = \frac{\sin x°}{6}$

$\sin y° = \frac{5 \sin x°}{6}$

$y° = \sin^{-1}\left(\frac{5 \sin x°}{6}\right) = 56.25\ldots°$

$y = 56.3$ (3 s.f.)
Using angle sum for a triangle
$z° = 180° - (x + y)° = 29.926\ldots°$
$z = 29.9$ (3 s.f.)

(f)



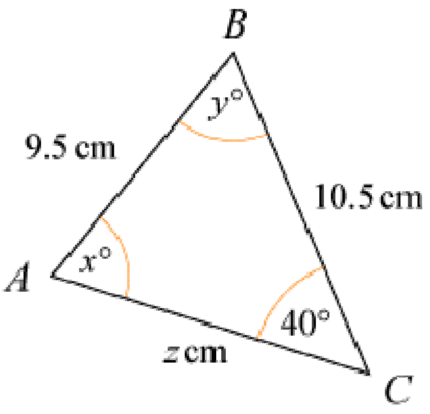
Using the line of symmetry through B

$\cos y° = \frac{6}{8}$

$y° = \cos^{-1}\left(\frac{3}{4}\right) = 41.40\ldots$

$y = 41.4$ (3 s.f.)
As triangle is isosceles
$z = y = 41.4$ (3 s.f.)
So $x° = 180° - (y + z)° = 97.2°$
$x = 97.2$ (3 s.f.)

(g)



Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin x°}{10.5} = \frac{\sin 40°}{9.5}$

$\sin x° = \frac{10.5 \sin 40°}{9.5}$

$x° = \sin^{-1}\left(\frac{10.5 \sin 40°}{9.5}\right)$ or $180° - \sin^{-1}\left(\frac{10.5 \sin 40°}{9.5}\right)$

$x° = 45.27°$ or $134.728\ldots°$
$x = 45.3$ (3 s.f.) or 135 (3 s.f.)

Using sine rule: $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{z}{\sin y°} = \frac{9.5}{\sin 40°}$

$z = \frac{9.5 \sin y°}{\sin 40°}$

When $x = 45.3$
$y° = 180° - (40 + 45.3)° = 94.7°$ so $y = 94.7$ (3 s.f.)

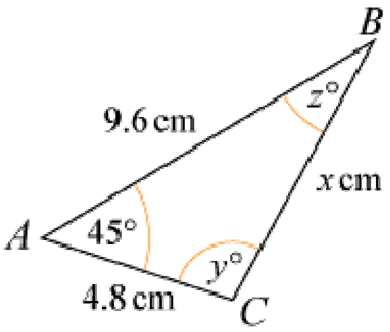
$z = \frac{9.5 \sin y°}{\sin 40°} = 14.7$ (3 s.f.)

When $x = 134.72\ldots°$
$y° = 180° - (40 + 134.72\ldots)° = 5.27° \Rightarrow y = 5.27$ (3 s.f.)

$z = \frac{9.5 \sin y°}{\sin 40°} = 1.36$ (3 s.f.)

So $x = 45.3$, $y = 94.7$, $z = 14.7$
or $x = 135$, $y = 5.27$, $z = 1.36$

(h)



Using $a^2 = b^2 + c^2 - 2bc \cos A$

$x^2 = 4.8^2 + 9.6^2 - 2 \times 4.8 \times 9.6 \times \cos 45° = 50.03 \ldots$

$x = 7.07$ (3 s.f.)

Using $\frac{\sin C}{c} = \frac{\sin A}{a}$ (As 9.6 > x, y > 45 and there are two possible values for y.)

$\frac{\sin y°}{9.6} = \frac{\sin 45°}{x}$

$\sin y° = \frac{9.6 \sin 45°}{x}$

$y° = \sin^{-1}\left(\frac{9.6 \sin 45°}{x}\right)$ or $180° - \sin^{-1}\left(\frac{9.6 \sin 45°}{x}\right)$

$y° = 73.67 \ldots°$ or $106.32 \ldots°$

$y = 73.7$ (3 s.f.) or 106 (3 s.f.)

When $y = 73.67 \ldots$

$z° = 180° - (45 + 73.67 \ldots)° = 61.32 \ldots°$

$z = 61.3$ (3 s.f.)

When $y = 106.32 \ldots$

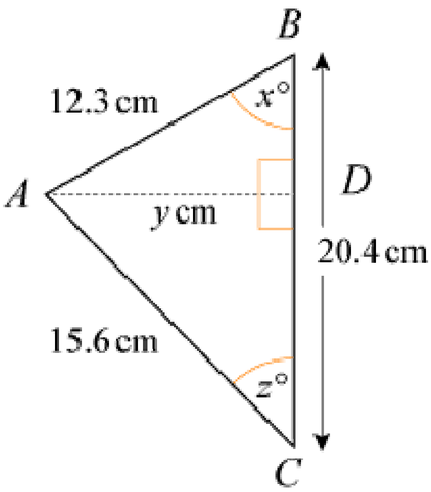
$z° = 180° - (45 + 106.32 \ldots)° = 28.67 \ldots°$

$z = 28.7$ (3 s.f.)

So $x = 7.07$, $y = 73.7$, $z = 61.3$

or $x = 7.07$, $y = 106$, $z = 28.7$

(i)



Using $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos x° = \frac{20.4^2 + 12.3^2 - 15.6^2}{2 \times 20.4 \times 12.3} = 0.6458 \ldots$

$x° = 49.77 \ldots°$

$x = 49.8$ (3 s.f.)

In the right-angled $\triangle ABD$

$\sin x° = \frac{y}{12.3} \Rightarrow y = 12.3 \sin x° = 9.39$ (3 s.f.)

In right-angled $\triangle ACD$

$\sin z° = \frac{y}{15.6} = 0.60199 \ldots$

$z° = 37.01 \ldots °$

$z = 37.0$ (3 s.f.)

So $x = 49.8$, $y = 9.39$, $z = 37.0$

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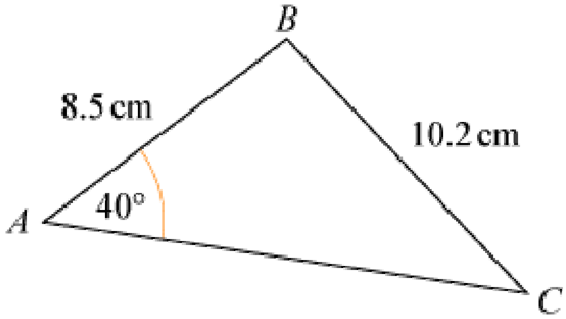
The sine and cosine rule
Exercise F, Question 2

Question:

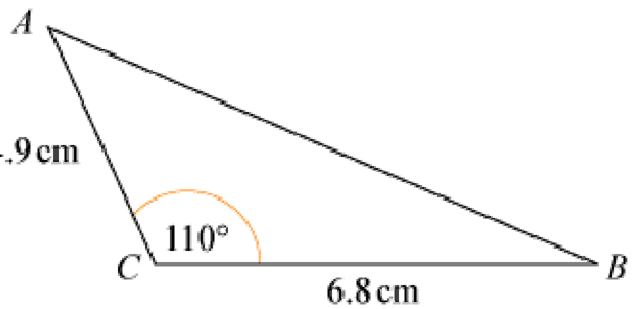
(*Note:* Try to use the neatest method, and give answers to 3 significant figures.)

Calculate the size of the remaining angles and the length of the third side in the following triangles:

(a)

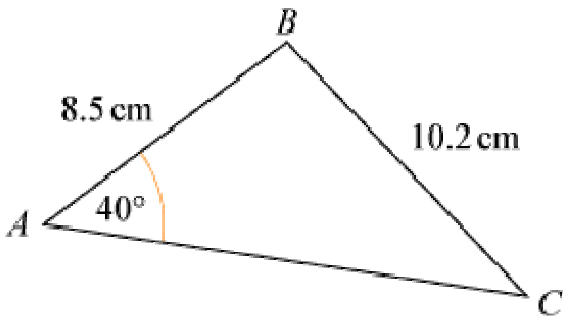


(b)



Solution:

(a)



Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin C}{8.5} = \frac{\sin 40°}{10.2}$

$\sin C = \frac{8.5 \sin 40°}{10.2}$

$C = \sin^{-1}\left(\frac{8.5 \sin 40°}{10.2}\right) = 32.388 \ldots °$

$C = 32.4°$ (3 s.f.)

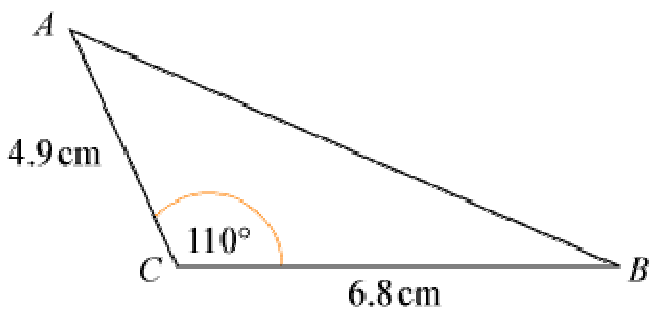
Angle $B = 180° - (40 + C)° = 107.6 \ldots °$

$B = 108°$ (3 s.f.)

Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$b = \frac{10.2 \sin B}{\sin 40°} = 15.1$ cm (3 s.f.)

(b)



Using $c^2 = a^2 + b^2 - 2ab \cos C$

$AB^2 = 6.8^2 + 4.9^2 - 2 \times 6.8 \times 4.9 \times \cos 110° = 93.04 \ldots$

AB = 9.6458 … = 9.65 cm (3 s.f.)

Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\sin A = \frac{6.8 \sin 110°}{AB} = 0.66245 \ldots$

$A = 41.49° = 41.5°$ (3 s.f.)

So $B = 180° - (110 + A)° = 28.5°$ (3 s.f.)

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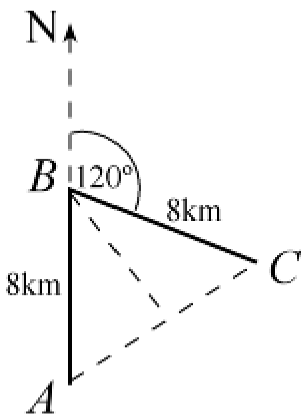
The sine and cosine rule
Exercise F, Question 3

Question:

(*Note:* Try to use the neatest method, and give answers to 3 significant figures.)

A hiker walks due north from *A* and after 8 km reaches *B*. She then walks a further 8 km on a bearing of 120° to *C*. Work out (a) the distance from *A* to *C* and (b) the bearing of *C* from *A*.

Solution:



(a) $\angle$ ABC = 180 ° − 120 ° = 60 °
As $\angle A = \angle C$, all angles are 60°; it is an equilateral triangle.
So AC = 8 km.

(b) As $\angle$ BAC = 60 ° ,
the bearing of *C* from *A* is 060°.

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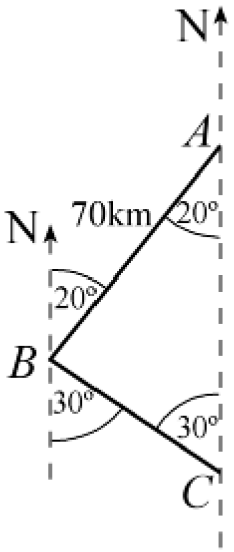
The sine and cosine rule
Exercise F, Question 4

Question:

(*Note:* Try to use the neatest method, and give answers to 3 significant figures.)

A helicopter flies on a bearing of 200° from *A* to *B*, where AB = 70 km. It then flies on a bearing of 150° from *B* to *C*, where *C* is due south of *A*. Work out the distance of *C* from *A*.

Solution:



From the diagram $\angle ABC = 180° - (20 + 30)° = 130°$

Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{AC}{\sin 130°} = \frac{70}{\sin 30°}$

$AC = \frac{70 \sin 130°}{\sin 30°} = 107.246 \ldots$

AC = 107 km (3 s.f.)

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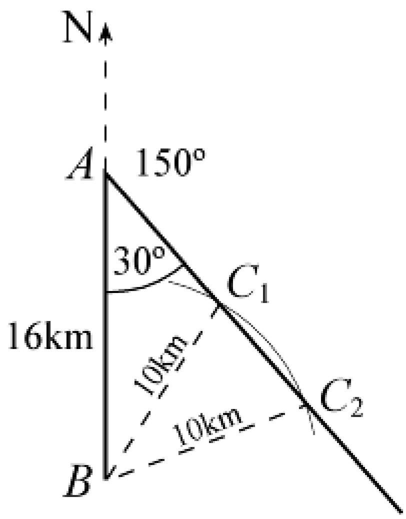
The sine and cosine rule
Exercise F, Question 5

Question:

(*Note:* Try to use the neatest method, and give answers to 3 significant figures.)

Two radar stations *A* and *B* are 16 km apart and *A* is due north of *B*. A ship is known to be on a bearing of 150° from *A* and 10 km from *B*. Show that this information gives two positions for the ship, and calculate the distance between these two positions.

Solution:



Using the sine rule: $\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin C}{16} = \frac{\sin 30°}{10}$

$\sin C = \frac{16 \sin 30°}{10} = 0.8$

$C = \sin^{-1}(0.8)$ or $180° - \sin^{-1}(0.8)$
$C = 53.1°$ or $126.9°$
$\angle AC_2B = 53.1°$, $\angle AC_1B = 127°$ (3 s.f.)
(Store the correct values; these are not required answers.)
Triangle BC_1C_2 is isosceles, so C_1C_2 can be found using this triangle, without finding AC_1 and AC_2.
Use the line of symmetry through *B*:

$\cos \angle C_1C_2B = \frac{\frac{1}{2}C_1C_2}{10}$

$\Rightarrow C_1C_2 = 20 \cos \angle C_1C_2B = 20 \cos \angle AC_2B = 20 \cos 53.1°$
$\Rightarrow C_1C_2 = 12$ km

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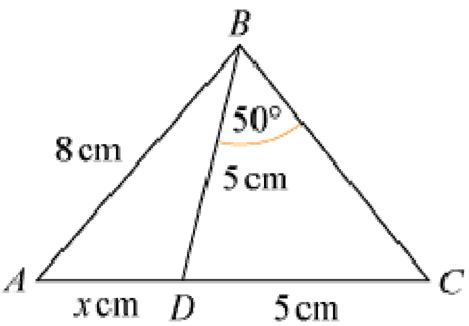
The sine and cosine rule
Exercise F, Question 6

Question:

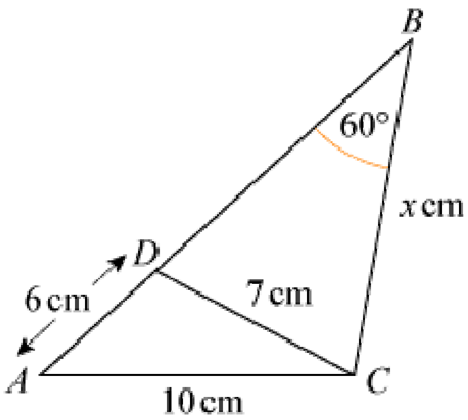
(*Note:* Try to use the neatest method, and give answers to 3 significant figures.)

Find x in each of the following diagrams:

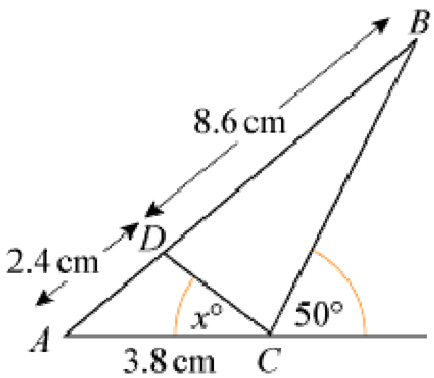
(a)



(b)

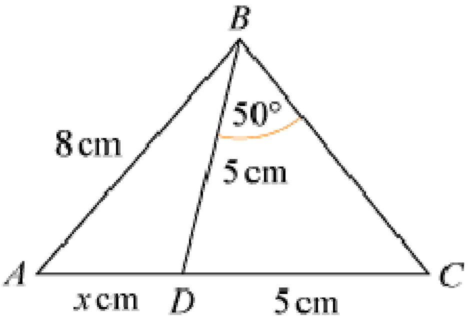


(c)



Solution:

(a)



In the isosceles $\triangle BDC$: $\angle$ BDC = 180 ° − (50 + 50) ° = 80 °

So $\angle$ BDA = 180 ° − 80 ° = 100 °

Using the sine rule in $\triangle ABD$: $\frac{\sin A}{a} = \frac{\sin D}{d}$

$\Rightarrow \frac{\sin A}{5} = \frac{\sin 100°}{8}$

$\Rightarrow \sin A = \frac{5 \sin 100°}{8}$

So $A = \sin^{-1}\left(\frac{5 \sin 100°}{8}\right) = 37.9886\ \ldots$

Angle ABD = 180 ° − (100 + A) ° = 42.01 … °

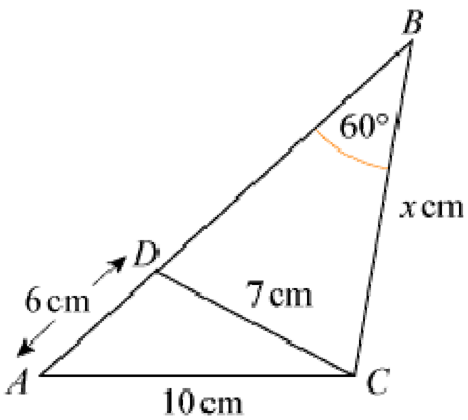
Using $\frac{b}{\sin B} = \frac{d}{\sin D}$

$\frac{x}{\sin B} = \frac{8}{\sin 100}$

$x = \frac{8 \sin B}{\sin 100°} = 5.436\ \ldots$

x = 5.44 (3 s.f.)

(b)



In $\triangle ADC$, using $\cos A = \frac{c^2 + d^2 - a^2}{2cd}$

$\cos A = \frac{6^2 + 10^2 - 7^2}{2 \times 6 \times 10} = 0.725$

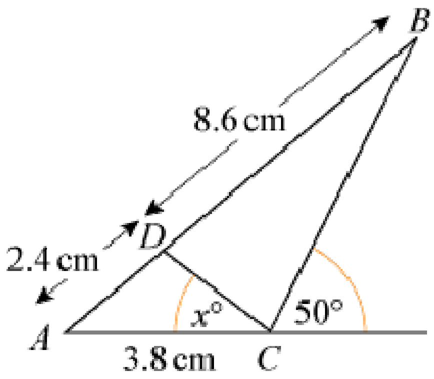
So A = 43.53 … °

Using the sine rule in $\triangle ABC$: $\frac{a}{\sin A} = \frac{b}{\sin B}$

So $\frac{x}{\sin A} = \frac{10}{\sin 60°}$

$\Rightarrow \quad x = \dfrac{10 \sin A}{\sin 60°} = 7.95$ (3 s.f.)

(c)



In $\triangle ABC$, $c = 11$ cm, $b = 3.8$ cm, $\angle ACB = 130°$

Using $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\sin B = \dfrac{3.8 \sin 130°}{11} = 0.2646 \ldots$

$B = 15.345 \ldots°$

So $A = 180° - (130 + B)° = 34.654 \ldots°$

In $\triangle ADC$, $c = 2.4$ cm, $d = 3.8$ cm, $A = 34.654 \ldots°$

Using the cosine rule: $a^2 = c^2 + d^2 - 2cd \cos A$

So $DC^2 = 2.4^2 + 3.8^2 - 2 \times 2.4 \times 3.8 \times \cos A = 5.1959 \ldots$

$\Rightarrow \quad DC = 2.279 \ldots$ cm.

Using the sine rule: $\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

$\sin x° = \dfrac{2.4 \sin A}{DC} = 0.59869 \ldots$

$x = 36.8$ (3 s.f.)

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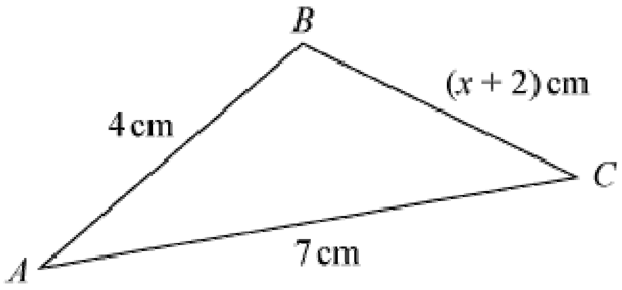
The sine and cosine rule
Exercise F, Question 7

Question:

(*Note:* Try to use the neatest method, and give answers to 3 significant figures.)

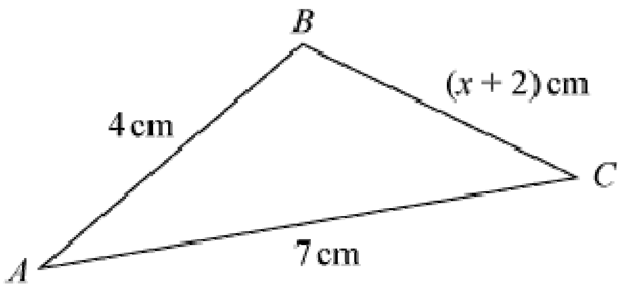
In $\triangle ABC$, shown below, AB = 4 cm, BC = $(x + 2)$ cm and AC = 7 cm.



(a) Explain how you know that $1 < x < 9$.

(b) Work out the value of x for the cases when
(i) $\angle ABC = 60°$ and
(ii) $\angle ABC = 45°$, giving your answers to 3 significant figures.

Solution:



(a) As AB + BC > AC
$4 + (x + 2) > 7$
$\Rightarrow \quad x + 2 > 3$
$\Rightarrow \quad x > 1$
As AB + AC > BC
$4 + 7 > x + 2$
$\Rightarrow \quad 9 > x$
So $1 < x < 9$

(b) Using $b^2 = a^2 + c^2 - 2ac \cos B$
(i) $7^2 = (x + 2)^2 + 4^2 - 2 \times (x + 2) \times 4 \times \cos 60°$
$49 = x^2 + 4x + 4 + 16 - 4(x + 2)$
$49 = x^2 + 4x + 4 + 16 - 4x - 8$
So $x^2 = 37$
$\Rightarrow \quad x = 6.08$ (3 s.f.)
(ii) $7^2 = (x + 2)^2 + 4^2 - 2 \times (x + 2) \times 4 \times \cos 45°$
$49 = x^2 + 4x + 4 + 16 - (8 \cos 45°) x - 16 \cos 45°$

So $x^2 + (4 - 8\cos 45°)x - (29 + 16\cos 45°) = 0$
or $x^2 + 4(1 - \sqrt{2})x - (29 + 8\sqrt{2}) = 0$

Use the quadratic equation formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with

$a = 1$
$b = 4 - 8\cos 45° = 4(1 - \sqrt{2}) = -1.6568 \ldots$
$c = -(29 + 16\cos 45°) = -(29 + 8\sqrt{2}) = -40.313 \ldots$
$x = 7.23$ (3 s.f.) (The other value of x is less than -2.)

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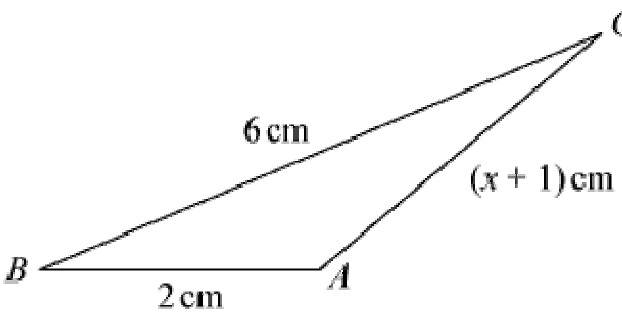
The sine and cosine rule
Exercise F, Question 8

Question:

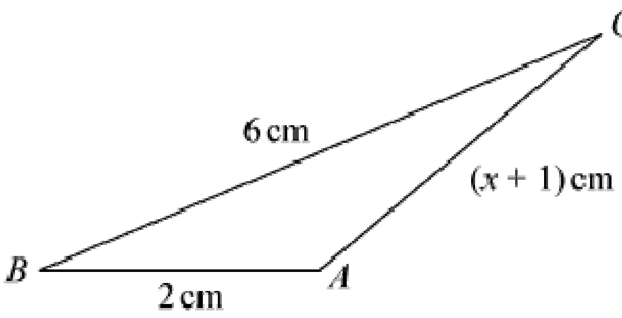
(*Note:* Try to use the neatest method, and give answers to 3 significant figures.)

In the triangle shown below, cos $\angle$ ABC = $\frac{5}{8}$.

Calculate the value of x.



Solution:



Using $b^2 = a^2 + c^2 - 2ac \cos B$ where $\cos B = \frac{5}{8}$

$(x + 1)^2 = 6^2 + 2^2 - 2 \times 6 \times 2 \times \frac{5}{8}$

$x^2 + 2x + 1 = 36 + 4 - 15$
$x^2 + 2x - 24 = 0$
$(x + 6) (x - 4) = 0$
So $x = 4 \quad (x > - 1)$

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Edexcel Modular Mathematics for AS and A-Level

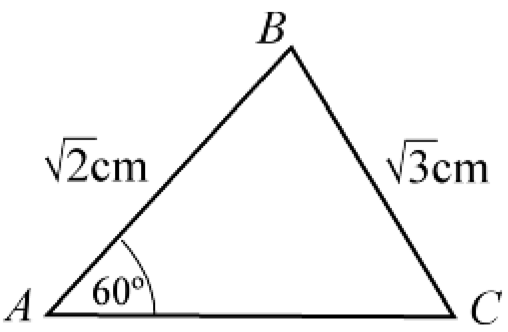
The sine and cosine rule
Exercise F, Question 9

Question:

(*Note:* Try to use the neatest method, and give answers to 3 significant figures.)

In $\triangle ABC$, AB = $\sqrt{2}$ cm, BC = $\sqrt{3}$ cm and $\angle BAC = 60°$. Show that $\angle ACB = 45°$ and find *AC*.

Solution:



Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

$\sin C = \frac{\sqrt{2} \sin 60°}{\sqrt{3}} = 0.7071 \quad \ldots$

$C = \sin^{-1} \left(\frac{\sqrt{2} \sin 60°}{\sqrt{3}} \right) = 45°$

$B = 180° - (60 + 45)° = 75°$

Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{AC}{\sin 75°} = \frac{\sqrt{3}}{\sin 60°}$

So $AC = \frac{\sqrt{3} \sin 75°}{\sin 60°} = 1.93$ cm (3 s.f.)

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Edexcel Modular Mathematics for AS and A-Level

The sine and cosine rule
Exercise F, Question 10

Question:

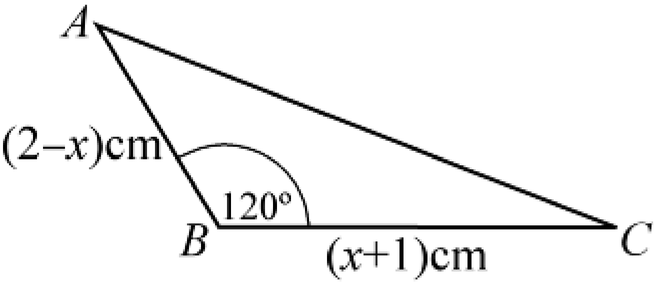
(*Note:* Try to use the neatest method, and give answers to 3 significant figures.)

In $\triangle ABC$, AB = $(2-x)$ cm, BC = $(x+1)$ cm and $\angle$ ABC = 120 ° :

(a) Show that $AC^2 = x^2 - x + 7$.

(b) Find the value of x for which AC has a minimum value.

Solution:



(a) Using the cosine rule: $b^2 = a^2 + c^2 - 2ac\ \cos\ B$
$AC^2 = (x+1)^2 + (2-x)^2 - 2(x+1)(2-x)\ \cos\ 120°$
$AC^2 = (x^2 + 2x + 1) + (4 - 4x + x^2) + (x+1)(2-x)$
$AC^2 = x^2 + 2x + 1 + 4 - 4x + x^2 - x^2 + 2x - x + 2$
$AC^2 = x^2 - x + 7$

(b) Using the method of completing the square:
$$x^2 - x + 7 \equiv \left(x - \frac{1}{2}\right)^2 + 7 - \frac{1}{4} \equiv \left(x - \frac{1}{2}\right)^2 + 6\frac{3}{4}$$
This is a minimum when $x - \frac{1}{2} = 0$, i.e. $x = \frac{1}{2}$.

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Edexcel Modular Mathematics for AS and A-Level

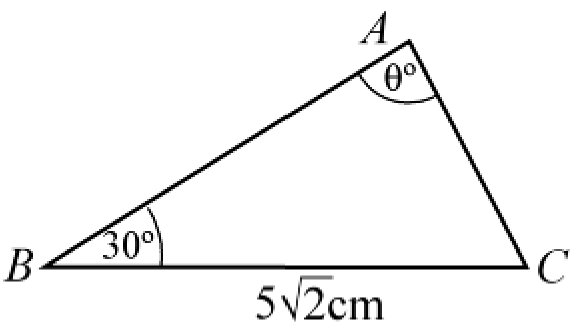
The sine and cosine rule
Exercise F, Question 11

Question:

(*Note:* Try to use the neatest method, and give answers to 3 significant figures.)

Triangle *ABC* is such that BC = $5\sqrt{2}$ cm, $\angle$ ABC = 30 ° and $\angle$ BAC = θ, where $\sin\theta = \frac{\sqrt{5}}{8}$. Work out the length of *AC*, giving your answer in the form $a\sqrt{b}$, where *a* and *b* are integers.

Solution:



Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$\frac{AC}{\sin 30°} = \frac{5\sqrt{2}}{\sin\theta°}$$

$$AC = \frac{5\sqrt{2}\sin 30°}{\left(\frac{\sqrt{5}}{8}\right)}$$

$$AC = \frac{5\sqrt{2}\sin 30° \times 8}{\sqrt{5}} = \left(\sqrt{5}\sqrt{2}\right)\left(8\sin 30°\right) = 4\sqrt{10}$$

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Edexcel Modular Mathematics for AS and A-Level

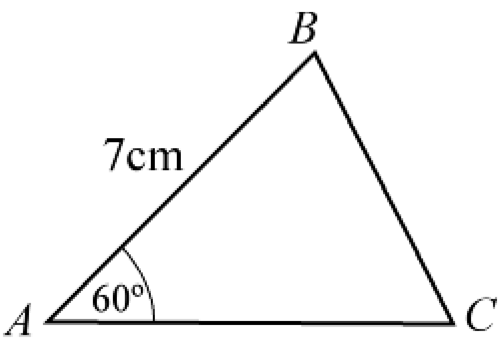
The sine and cosine rule
Exercise F, Question 12

Question:

(*Note:* Try to use the neatest method, and give answers to 3 significant figures.)

The perimeter of $\triangle ABC$ = 15 cm. Given that AB = 7 cm and $\angle$ BAC = 60 ° , find the lengths *AC* and *BC*.

Solution:



Using the cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$
with $a = x$, $b = (8 - x)$, $c = 7$ and $A = 60°$

$x^2 = (8 - x)^2 + 7^2 - 2(8 - x) \times 7 \times \cos 60°$

$x^2 = 64 - 16x + x^2 + 49 - 7(8 - x)$

$x^2 = 64 - 16x + x^2 + 49 - 56 + 7x$

$\Rightarrow \quad 9x = 57$

$\Rightarrow \quad x = \frac{57}{9} = \frac{19}{3} = 6\frac{1}{3}$

So BC = $6\frac{1}{3}$ cm and AC = $\left(8 - 6\frac{1}{3}\right)$ cm $= 1\frac{2}{3}$ cm.

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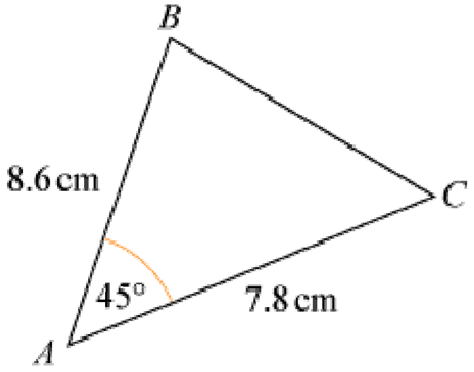
The sine and cosine rule
Exercise G, Question 1

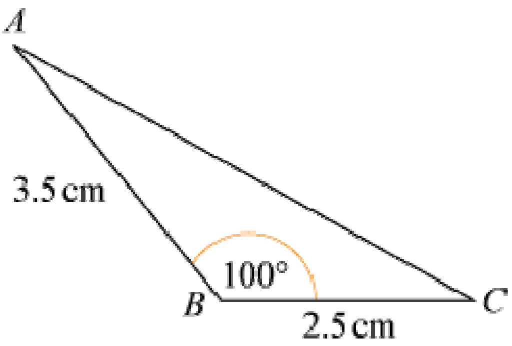
Question:

Calculate the area of the following triangles:

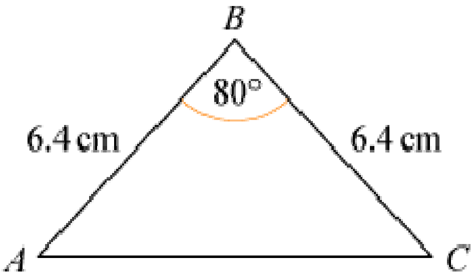
(a)



(b)

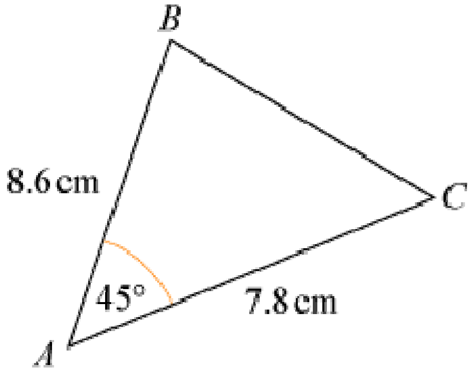


(c)



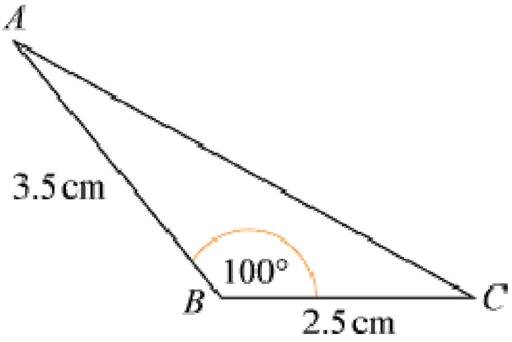
Solution:

(a)



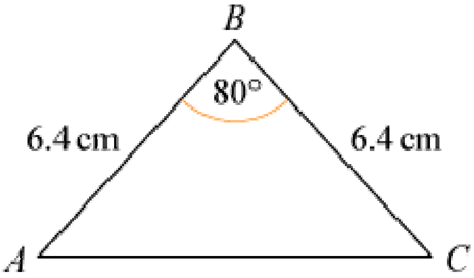
Area $= \frac{1}{2} \times 7.8 \times 8.6 \times \sin 45° = 23.71 \ldots = 23.7 \text{ cm}^2$ (3 s.f.)

(b)



Area $= \frac{1}{2} \times 2.5 \times 3.5 \times \sin 100° = 4.308 \ldots = 4.31 \text{ cm}^2$ (3 s.f.)

(c)



Area $= \frac{1}{2} \times 6.4 \times 6.4 \times \sin 80° = 20.16 \ldots = 20.2 \text{ cm}^2$ (3 s.f.)

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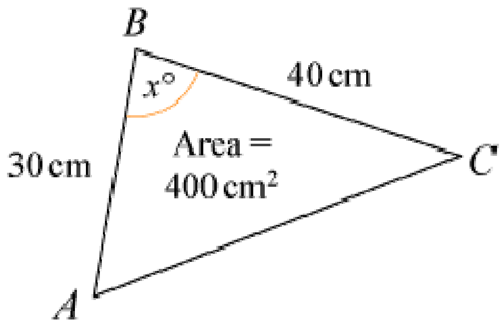
The sine and cosine rule
Exercise G, Question 2

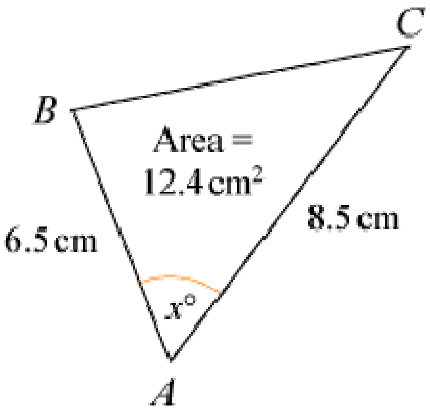
Question:

Work out the possible values of x in the following triangles:

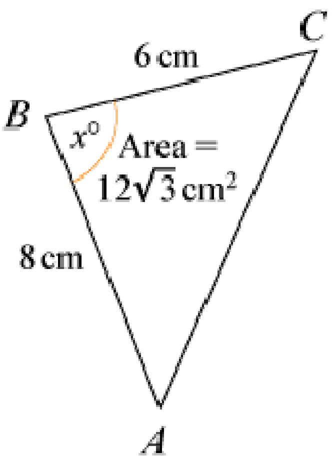
(a)



(b)

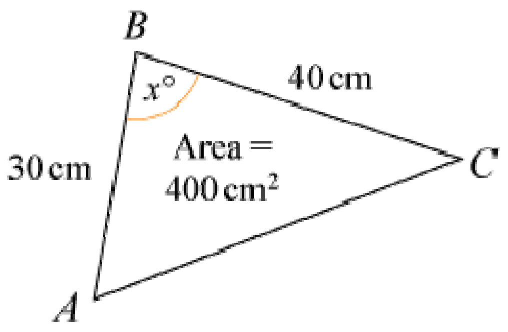


(c)



Solution:

(a)



Using area $= \frac{1}{2}ac \sin B$

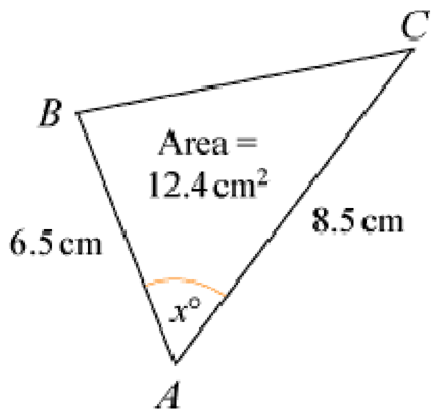
$400 = \frac{1}{2} \times 40 \times 30 \times \sin x°$

So $\sin x° = \frac{400}{600} = \frac{2}{3}$

$x° = \sin^{-1}\left(\frac{2}{3}\right)$ or $180° - \sin^{-1}\left(\frac{2}{3}\right)$

$x = 41.8$ (3 s.f.) or 138 (3 s.f.)

(b)



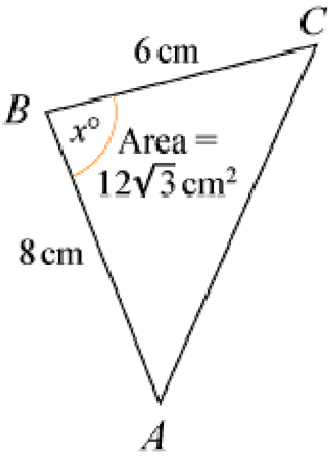
Using area $= \frac{1}{2}bc \sin A$

$12.4 = \frac{1}{2} \times 8.5 \times 6.5 \times \sin x°$

So $\sin x° = \frac{12.4}{27.625} = 0.04488 \quad \ldots$

$x = 26.7$ (3 s.f.) or 153 (3 s.f.)

(c)



Using area $= \frac{1}{2}ac \sin B$

$12\sqrt{3} = \frac{1}{2} \times 6 \times 8 \sin x°$

So $\sin x° = \frac{12\sqrt{3}}{24} = \frac{\sqrt{3}}{2}$

$x = 60$ or 120

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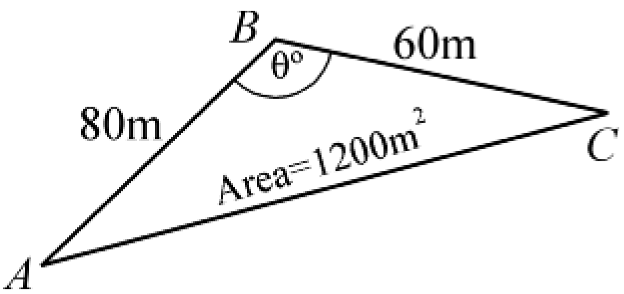
Edexcel Modular Mathematics for AS and A-Level

The sine and cosine rule
Exercise G, Question 3

Question:

A fenced triangular plot of ground has area 1200 m^2. The fences along the two smaller sides are 60 m and 80 m respectively and the angle between them is $\theta°$. Show that $\theta = 150$, and work out the total length of fencing.

Solution:



Using area $= \frac{1}{2} ac \sin B$

$1200 = \frac{1}{2} \times 60 \times 80 \times \sin \theta°$

$\sin \theta° = \frac{1200}{2400} = \frac{1}{2}$

$\theta = 30$ or 150
but as AC is the largest side, θ must be the largest angle.
So $\theta = 150$

Using the cosine rule: $b^2 = a^2 + c^2 - 2ac \cos B$ to find AC
$AC^2 = 60^2 + 80^2 - 2 \times 60 \times 80 \times \cos 150° = 18313.84 \ldots$
$AC = 135.3 \ldots$
$AC = 135$ m (3 s.f.)
So perimeter $= 60 + 80 + 135 = 275$ m (3 s.f.)

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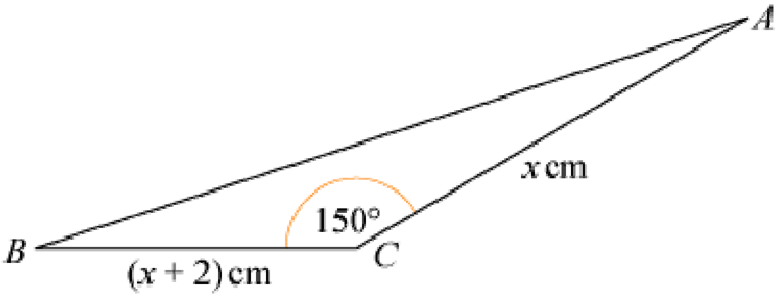
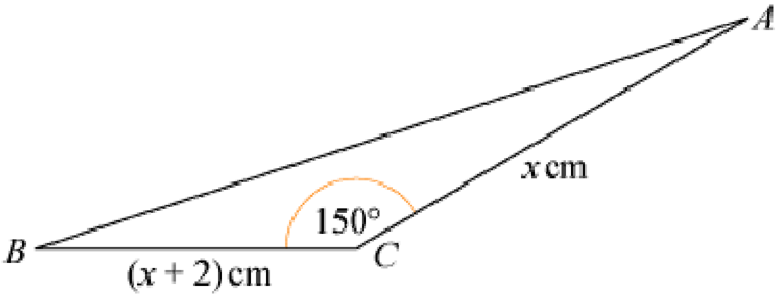
The sine and cosine rule
Exercise G, Question 4

Question:

In triangle *ABC*, shown below, BC = $(x+2)$ cm, AC = x cm and $\angle$ BCA = 150°.
Given that the area of the triangle is 5 cm^2, work out the value of x, giving your answer to 3 significant figures.



Solution:



Area of $\triangle ABC = \frac{1}{2}x\left(x+2\right)\sin 150° \text{ cm}^2$

So $5 = \frac{1}{2}x\left(x+2\right) \times \frac{1}{2}$

So $20 = x(x+2)$
or $x^2 + 2x - 20 = 0$

Using the quadratic equation formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-2 \pm \sqrt{84}}{2} = 3.582 \ldots$ or $-5.582 \ldots$

As $x > 0$, $x = 3.58$ (3 s.f.)

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The sine and cosine rule
Exercise G, Question 5

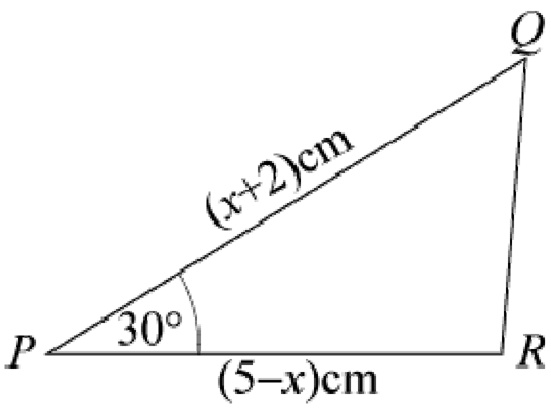
Question:

In $\triangle PQR$, PQ = $(x + 2)$ cm, PR = $(5 - x)$ cm and $\angle QPR = 30°$.
The area of the triangle is A cm^2:

(a) Show that $A = \frac{1}{4} \left(10 + 3x - x^2 \right)$.

(b) Use the method of completing the square, or otherwise, to find the maximum value of A and give the corresponding value of x.

Solution:



(a) Using area of $\triangle PQR = \frac{1}{2} qr \sin P$

$$A \text{ cm}^2 = \frac{1}{2} \left(5 - x \right) \left(x + 2 \right) \sin 30° \text{ cm}^2$$

$$\Rightarrow A = \frac{1}{2} \left(5x - 2x + 10 - x^2 \right) \times \frac{1}{2}$$

$$\Rightarrow A = \frac{1}{4} \left(10 + 3x - x^2 \right)$$

(b) $10 + 3x - x^2$

$= - (x^2 - 3x - 10)$

$= - \left[\left(x - 1\frac{1}{2} \right)^2 - 2\frac{1}{4} - 10 \right]$ (completing the square)

$= - \left[\left(x - 1\frac{1}{2} \right)^2 - 12\frac{1}{4} \right]$

$= 12\frac{1}{4} - \left(x - 1\frac{1}{2} \right)^2$

The maximum value of $10 + 3x - x^2 = 12\frac{1}{4}$, when $x = 1\frac{1}{2}$.

The maximum value of A is $\frac{1}{4}\left(12\frac{1}{4}\right) = 3\frac{1}{16}$, when $x = 1\frac{1}{2}$.

(You could find the maximum using differentiation.)

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Edexcel Modular Mathematics for AS and A-Level

The sine and cosine rule
Exercise G, Question 6

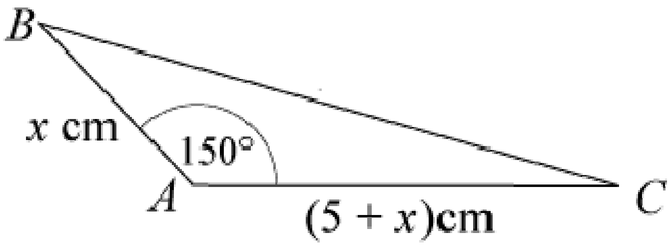
Question:

In $\triangle ABC$, AB = x cm, AC = $(5 + x)$ cm and $\angle$ BAC = 150°. Given that the area of the triangle is $3\frac{3}{4}$ cm^2:

(a) Show that x satisfies the equation $x^2 + 5x - 15 = 0$.

(b) Calculate the value of x, giving your answer to 3 significant figures.

Solution:



(a) Using area of $\triangle BAC = \frac{1}{2}bc\ \sin\ A$

$3\frac{3}{4}\ \text{cm}^2 = \frac{1}{2}x\left(5 + x\right)\ \sin\ 150°\ \ \text{cm}^2$

$3\frac{3}{4} = \frac{1}{2}\left[5x + x^2\right] \times \frac{1}{2}$

$\Rightarrow\ \ 15 = 5x + x^2$

$\Rightarrow\ \ x^2 + 5x - 15 = 0$

(b) Using the quadratic equation formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-5 \pm \sqrt{85}}{2} = 2.109\ \ldots$ or $-7.109\ \ldots$

As $x > 0$, $x = 2.11$ (3 s.f.)

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Edexcel Modular Mathematics for AS and A-Level

The sine and cosine rule
Exercise H, Question 1

Question:

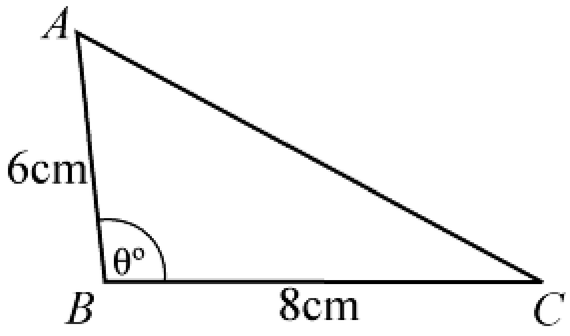
(Given non-exact answers to 3 significant figures.)

The area of a triangle is 10 cm^2. The angle between two of the sides, of length 6 cm and 8 cm respectively, is obtuse. Work out:

(a) The size of this angle.

(b) The length of the third side.

Solution:



(a) Using area of $\triangle ABC = \frac{1}{2}ac \sin B$

$10 \text{ cm}^2 = \frac{1}{2} \times 6 \times 8 \times \sin \theta° \text{ cm}^2$

So $10 = 24 \sin \theta°$

So $\sin \theta° = \frac{10}{24} = \frac{5}{12}$

$\Rightarrow \theta = 24.6$ or 155 (3 s.f.)

As θ is obtuse, $\angle ABC = 155°$ (3 s.f.)

(b) Using the cosine rule: $b^2 = a^2 + c^2 - 2ac \cos B$
$AC^2 = 8^2 + 6^2 - 2 \times 8 \times 6 \times \cos B = 187.26 \ldots$
$AC = 13.68 \ldots$
The third side has length 13.7m (3 s.f.)

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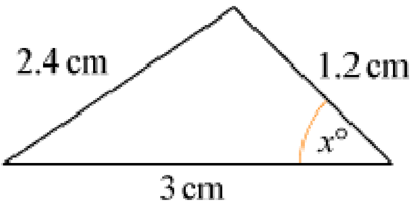
The sine and cosine rule
Exercise H, Question 2

Question:

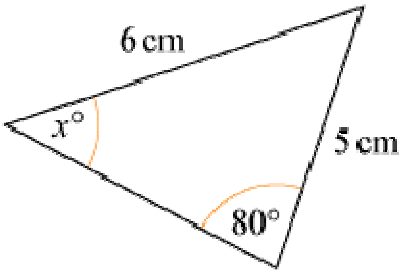
(Give non-exact answers to 3 significant figures.)

In each triangle below, find the value of *x* and the area of the triangle:

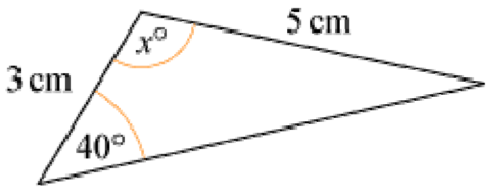
(a)



(b)

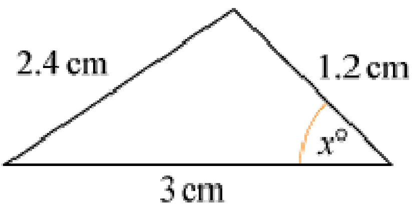


(c)



Solution:

(a)



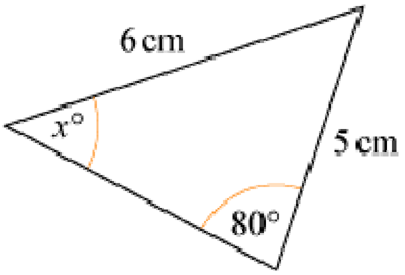
Using the cosine rule:

$\cos x° = \frac{3^2 + 1.2^2 - 2.4^2}{2 \times 3 \times 1.2} = 0.65$

$x = \cos^{-1}(0.65) = 49.458 \ldots$
$x = 49.5$ (3 s.f.)
Using the area of a triangle formula:

$\text{area} = \frac{1}{2} \times 1.2 \times 3 \times \sin x° \text{ cm}^2 = 1.367 \ldots \text{ cm}^2 = 1.37 \text{ cm}^2$ (3 s.f.)

(b)



Using the sine rule:

$$\frac{\sin x°}{5} = \frac{\sin 80°}{6}$$

$$\sin x° = \frac{5 \sin 80°}{6} = 0.8206 \quad \ldots$$

$x = 55.152 \quad \ldots$

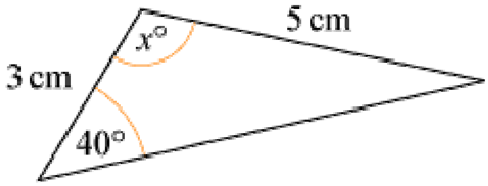
$x = 55.2$ (3 s.f.)

The angle between 5 cm and 6 cm sides is $180° - (80 + x)° = (100 - x)°$.

Using the area of a triangle formula:

$$\text{area} = \frac{1}{2} \times 5 \times 6 \times \sin\left(100 - x\right)° \text{ cm}^2 = 10.6 \text{ cm}^2 \text{ (3 s.f.)}$$

(c)



Using the sine rule to find angle opposite 3 cm. Call this $y°$.

$$\frac{\sin y°}{3} = \frac{\sin 40°}{5}$$

$$\sin y° = \frac{3 \sin 40°}{5}$$

$\Rightarrow \quad y = 22.68 \quad \ldots$

So $x = 180 - (40 + y) = 117.3 \quad \ldots \quad = 117$ (3 s.f.)

Area of triangle $= \frac{1}{2} \times 3 \times 5 \times \sin x° = 6.66 \text{ cm}^2$ (3 s.f.)

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

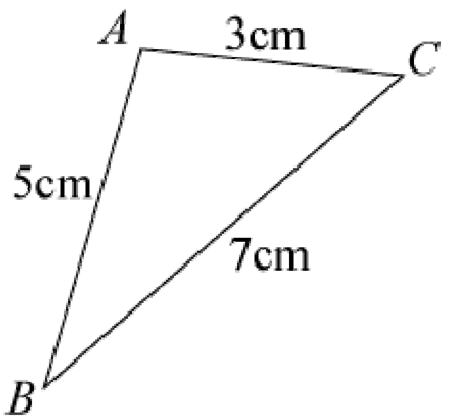
The sine and cosine rule
Exercise H, Question 3

Question:

(Give non-exact answers to 3 significant figures.)

The sides of a triangle are 3 cm, 5 cm and 7 cm respectively. Show that the largest angle is 120°, and find the area of the triangle.

Solution:



Using cosine rule to find angle A

$\cos A = \frac{3^2 + 5^2 - 7^2}{2 \times 3 \times 5} = -0.5$

$A = \cos^{-1}(-0.5) = 120°$

Area of triangle $= \frac{1}{2} \times 3 \times 5 \times \sin A$ cm^2 = 6.495 … cm^2 = 6.50 cm^2 (3 s.f.)

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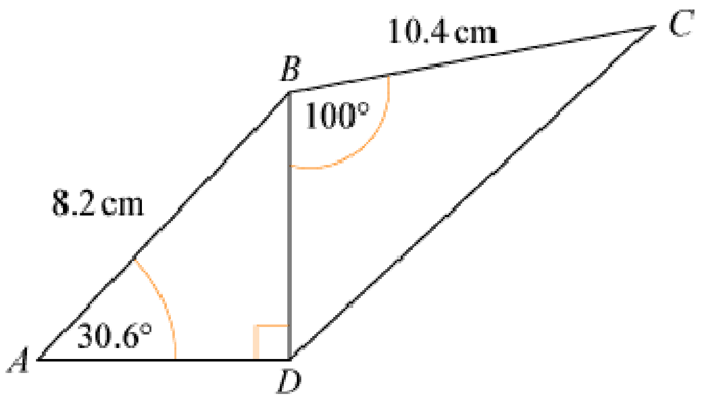
The sine and cosine rule
Exercise H, Question 4

Question:

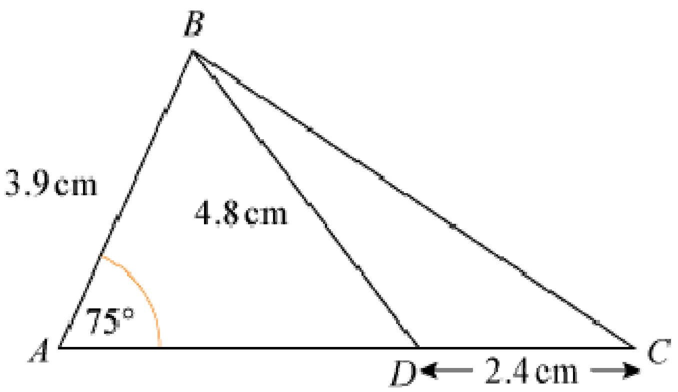
(Give non-exact answers to 3 significant figures.)

In each of the figures below calculate the total area:

(a)

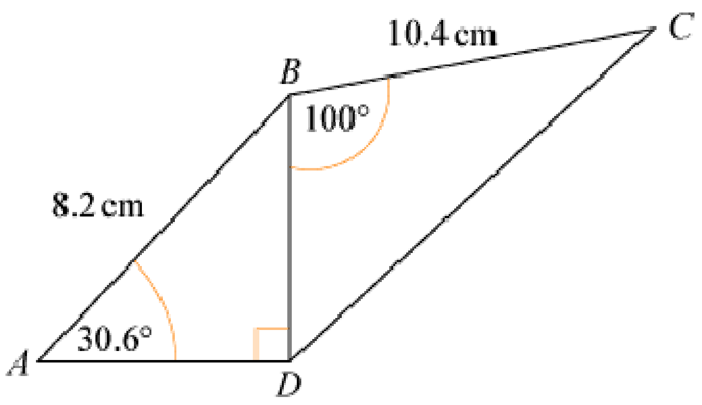


(b)



Solution:

(a)



In $\triangle BDA$: $\frac{BD}{8.2} = \sin 30.6°$

$\Rightarrow$ $BD = 8.2 \sin 30.6° = 4.174 \ldots$

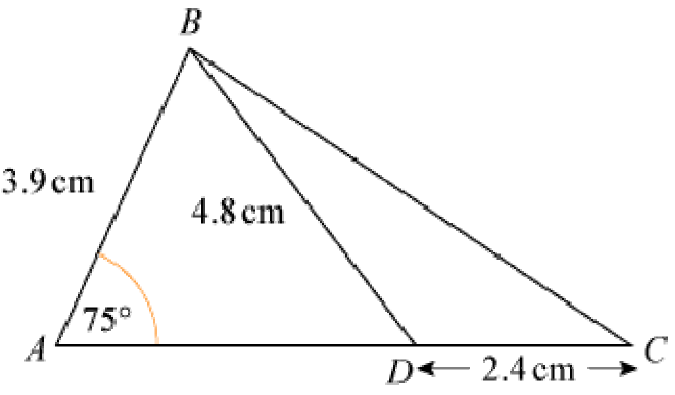
Angle $ABD = 90° - 30.6° = 59.4°$

Area of $\triangle ABD = \frac{1}{2} \times 8.2 \times \text{BD} \times \sin\ 59.4° = 14.7307 \ldots \text{ cm}^2$

Area of $\triangle BDC = \frac{1}{2} \times 10.4 \times \text{BD} \times \sin\ 100° = 21.375 \ldots \text{ cm}^2$

Total area = area of $\triangle ABD$ + area $\triangle BDC$ = 36.1 cm^2 (3 s.f.)

(b)



In $\triangle ABD$, using the sine rule to find $\angle$ ADB,

$$\frac{\sin\ \angle \text{ADB}}{3.9} = \frac{\sin\ 75°}{4.8}$$

$$\sin\ \angle \text{ADB} = \frac{3.9\ \sin\ 75°}{4.8}$$

$$\angle \text{ADB} = \sin^{-1}\left(\frac{3.9\ \sin\ 75°}{4.8}\right) = 51.7035 \ldots °$$

So $\angle \text{ABD} = 180° - (75 + \angle \text{ADB})° = 53.296 \ldots °$

Area of $\triangle ABD = \frac{1}{2} \times 3.9 \times 4.8 \times \sin\ \angle \text{ABD}\ \text{cm}^2 = 7.504 \ldots \text{ cm}^2$

In $\triangle BDC$, $\angle \text{BDC} = 180° - \angle \text{BDA} = 128.29 \ldots °$

Area of $\triangle BDC = \frac{1}{2} \times 2.4 \times 4.8 \times \sin\ \angle \text{BDC}\ \text{cm}^2 = 4.520 \ldots \text{ cm}^2$

Total area = area of $\triangle ABD$ + area of $\triangle BDC$ = 12.0 cm^2 (3 s.f.)

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The sine and cosine rule
Exercise H, Question 5

Question:

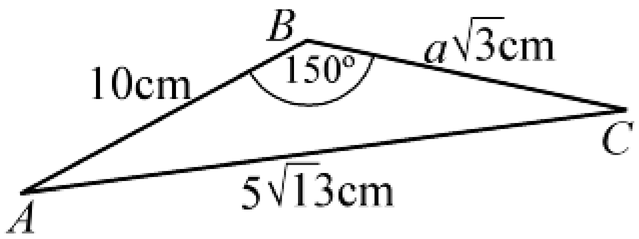
(Give non-exact answers to 3 significant figures.)

In $\triangle ABC$, AB = 10 cm, BC = $a\sqrt{3}$ cm, AC = $5\sqrt{13}$ cm and $\angle$ ABC = 150 ° . Calculate:

(a) The value of a.

(b) The exact area of $\triangle ABC$.

Solution:



(a) Using the cosine rule: $b^2 = a^2 + c^2 - 2ac\cos B$
$(5\sqrt{13})^2 = (a\sqrt{3})^2 + 10^2 - 2 \times a\sqrt{3} \times 10 \times \cos 150°$
$325 = 3a^2 + 100 + 30a$
$3a^2 + 30a - 225 = 0$
$a^2 + 10a - 75 = 0$
$(a + 15)(a - 5) = 0$
$\Rightarrow \quad a = 5$ as $a > 0$

(b) Area of $\triangle ABC = \frac{1}{2} \times 10 \times 5\sqrt{3} \times \sin 150° \text{ cm}^2 = 12.5\sqrt{3} \text{ cm}^2$

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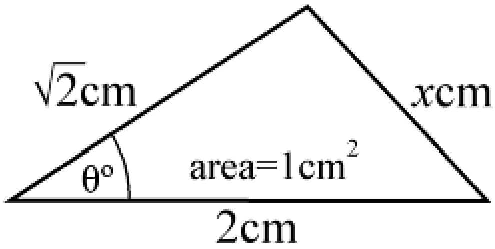
The sine and cosine rule
Exercise H, Question 6

Question:

(Give non-exact answers to 3 significant figures.)

In a triangle, the largest side has length 2 cm and one of the other sides has length $\sqrt{2}$ cm. Given that the area of the triangle is 1 cm^2, show that the triangle is right-angled and isosceles.

Solution:



Using the area formula:

$1 = \frac{1}{2} \times 2 \times \sqrt{2} \times \sin \theta°$

$\Rightarrow \sin \theta° = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 45$ or 135

but as θ is not the largest angle, θ must be 45.
Using the cosine rule to find x:
$x^2 = 2^2 + (\sqrt{2})^2 - 2 \times 2 \times \sqrt{2} \times \cos 45°$
$x^2 = 4 + 2 - 4 = 2$
So $x = \sqrt{2}$
So the triangle is isosceles with two angles of 45°.
It is a right-angled isosceles triangle.

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The sine and cosine rule
Exercise H, Question 7

Question:

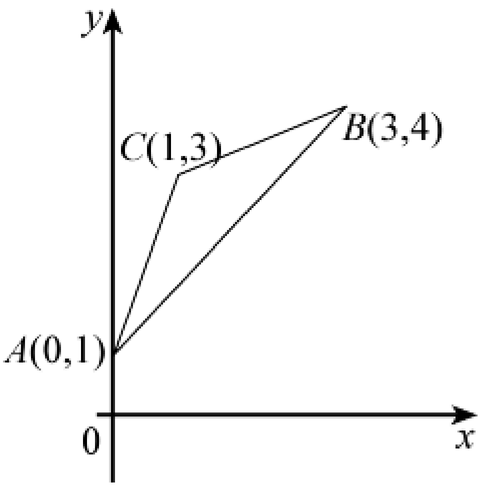
(Give non-exact answers to 3 significant figures.)

The three points *A*, *B* and *C*, with coordinates $A(0, 1)$, $B(3, 4)$ and $C(1, 3)$ respectively, are joined to form a triangle:

(a) Show that $\cos \angle ACB = -\frac{4}{5}$.

(b) Calculate the area of $\triangle ABC$.

Solution:



(a) $AC = \sqrt{(1-0)^2 + (3-1)^2} = \sqrt{5} = b$
$BC = \sqrt{(3-1)^2 + (4-3)^2} = \sqrt{5} = a$
$AB = \sqrt{(3-0)^2 + (4-1)^2} = \sqrt{18} = c$

Using the cosine rule: $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \frac{5 + 5 - 18}{2 \times \sqrt{5} \times \sqrt{5}} = \frac{-8}{10} = \frac{-4}{5}$

(b) Using the area formula:
area of $\triangle ABC = \frac{1}{2} ab \sin C = \frac{1}{2} \times \sqrt{5} \times \sqrt{5} \times \sin C = 1.5 \text{ cm}^2$

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The sine and cosine rule
Exercise H, Question 8

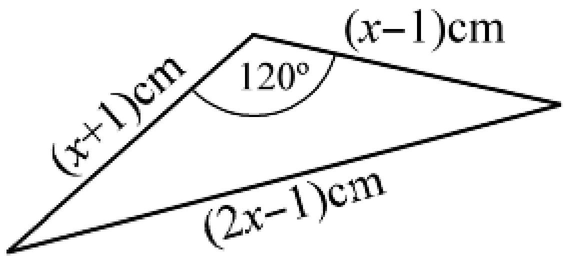
Question:

(Give non-exact answers to 3 significant figures.)

The longest side of a triangle has length $(2x-1)$ cm. The other sides have lengths $(x-1)$ cm and $(x+1)$ cm. Given that the largest angle is 120°, work out:

(a) the value of x and (b) the area of the triangle.

Solution:



(a) Using the cosine rule:
$(2x-1)^2 = (x+1)^2 + (x-1)^2 - 2(x+1)(x-1)\cos 120°$
$4x^2 - 4x + 1 = (x^2 + 2x + 1) + (x^2 - 2x + 1) + (x^2 - 1)$
$4x^2 - 4x + 1 = 3x^2 + 1$
$x^2 - 4x = 0$
$x(x-4) = 0$
$\Rightarrow \quad x = 4$ as $x > 1$

(b) Area of triangle

$= \frac{1}{2} \times \left(x+1 \right) \times \left(x-1 \right) \times \sin 120° \text{ cm}^2$

$= \frac{1}{2} \times 5 \times 3 \times \sin 120° \text{ cm}^2$

$= \frac{1}{2} \times 5 \times 3 \times \frac{\sqrt{3}}{2} \text{ cm}^2$

$= \frac{15\sqrt{3}}{4} \text{ cm}^2$

$= 6.50 \text{ cm}^2$ (3 s.f.)