Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sketching curves
Exercise A, Question 1

Question:

Sketch the following curves and indicate clearly the points of intersection with the axes:

(a) $y = (x - 3)(x - 2)(x + 1)$

(b) $y = (x - 1)(x + 2)(x + 3)$

(c) $y = (x + 1)(x + 2)(x + 3)$

(d) $y = (x + 1)(1 - x)(x + 3)$

(e) $y = (x - 2)(x - 3)(4 - x)$

(f) $y = x(x - 2)(x + 1)$

(g) $y = x(x + 1)(x - 1)$

(h) $y = x(x + 1)(1 - x)$

(i) $y = (x - 2)(2x - 1)(2x + 1)$

(j) $y = x(2x - 1)(x + 3)$

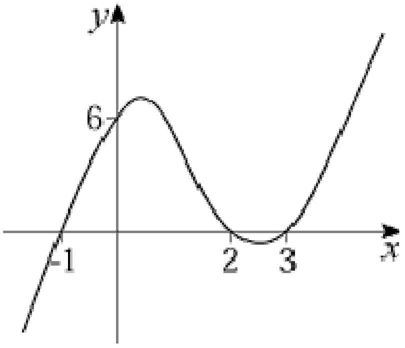
Solution:

(a) $y = 0 \Rightarrow x = -1, 2, 3$

$x = 0 \Rightarrow y = 6$

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$

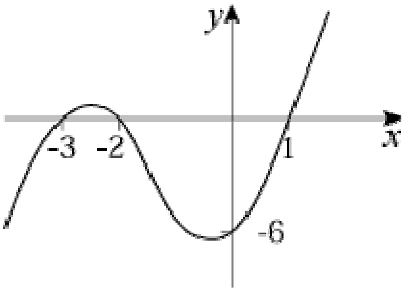


(b) $y = 0 \Rightarrow x = 1, -2, -3$

$x = 0 \Rightarrow y = -6$

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$

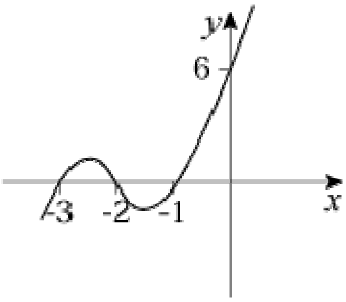


(c) $y = 0 \quad \Rightarrow \quad x = -1, -2, -3$

$x = 0 \quad \Rightarrow \quad y = 6$

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$

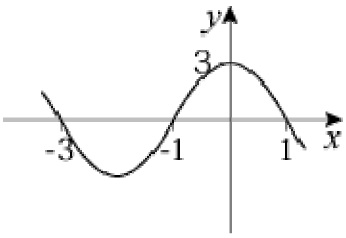


(d) $y = 0 \quad \Rightarrow \quad x = -1, 1, -3$

$x = 0 \quad \Rightarrow \quad y = 3$

$x \to \infty, y \to -\infty$

$x \to -\infty, y \to \infty$

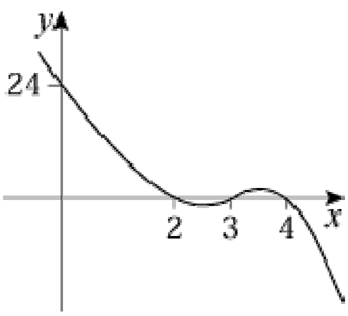


(e) $y = 0 \quad \Rightarrow \quad x = 2, 3, 4$

$x = 0 \quad \Rightarrow \quad y = 24$

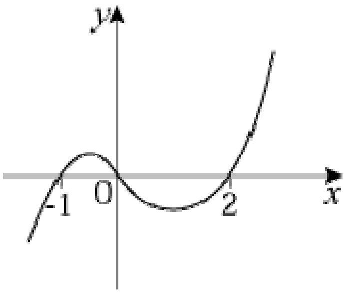
$x \to \infty, y \to -\infty$

$x \to -\infty, y \to \infty$

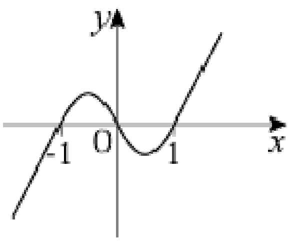


(f) $y = 0 \quad \Rightarrow \quad x = 0, -1, 2$

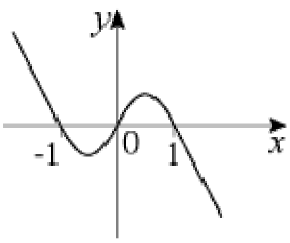
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



(g) $y = 0 \Rightarrow x = 0, -1, 1$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$

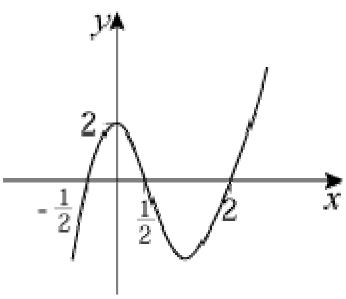


(h) $y = 0 \Rightarrow x = 0, -1, 1$
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



(i) $y = 0 \Rightarrow x = 2, \frac{1}{2}, -\frac{1}{2}$

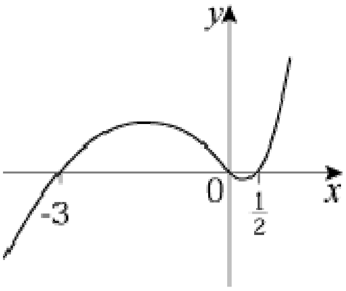
$x = 0 \Rightarrow y = 2$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



(j) $y = 0 \Rightarrow x = 0, \frac{1}{2}, -3$

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sketching curves
Exercise A, Question 2

Question:

Sketch the curves with the following equations:

(a) $y = (x + 1)^2(x - 1)$

(b) $y = (x + 2)(x - 1)^2$

(c) $y = (2 - x)(x + 1)^2$

(d) $y = (x - 2)(x + 1)^2$

(e) $y = x^2(x + 2)$

(f) $y = (x - 1)^2 x$

(g) $y = (1 - x)^2(3 + x)$

(h) $y = (x - 1)^2(3 - x)$

(i) $y = x^2(2 - x)$

(j) $y = x^2(x - 2)$

Solution:

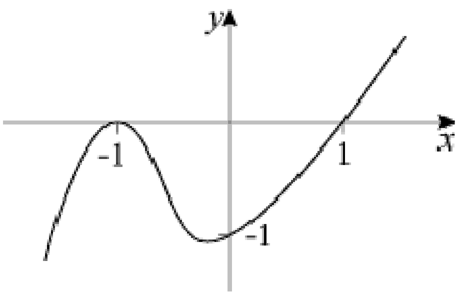
(a) $y = 0 \quad \Rightarrow \quad x = -1$ (twice), 1

$x = 0 \quad \Rightarrow \quad y = -1$

Turning point at $(-1, 0)$.

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



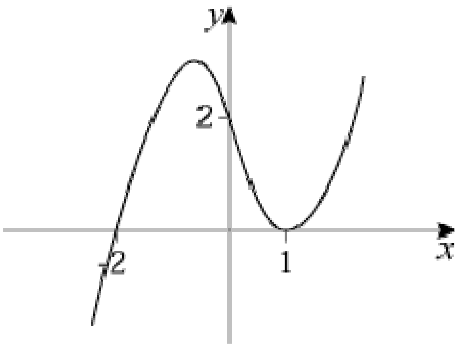
(b) $y = 0 \quad \Rightarrow \quad x = -2$, 1 (twice)

$x = 0 \quad \Rightarrow \quad y = 2$

Turning point at $(1, 0)$.

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



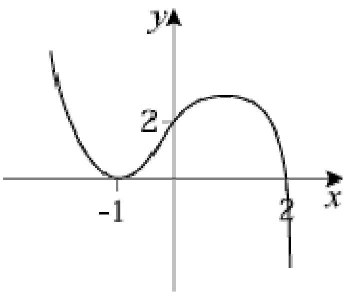
(c) $y = 0 \quad \Rightarrow \quad x = 2, \ -1$ (twice)

$x = 0 \quad \Rightarrow \quad y = 2$

Turning point at $(-1 , 0)$.

$x \to \infty , y \to -\infty$

$x \to -\infty , y \to \infty$



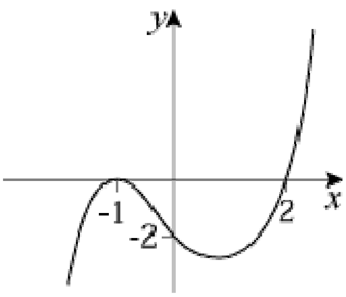
(d) $y = 0 \quad \Rightarrow \quad x = 2, \ -1$ (twice)

$x = 0 \quad \Rightarrow \quad y = -2$

Turning point at $(-1 , 0)$.

$x \to \infty , y \to \infty$

$x \to -\infty , y \to -\infty$

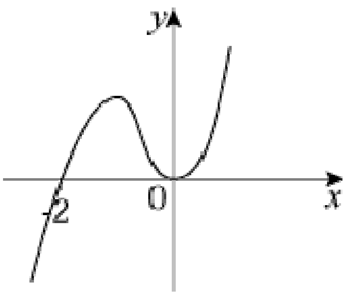


(e) $y = 0 \quad \Rightarrow \quad x = 0$ (twice), -2

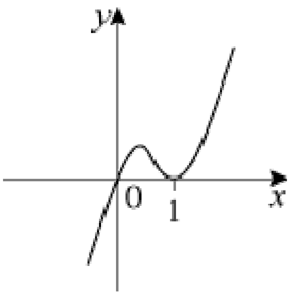
Turning point at $(0 , 0)$.

$x \to \infty , y \to \infty$

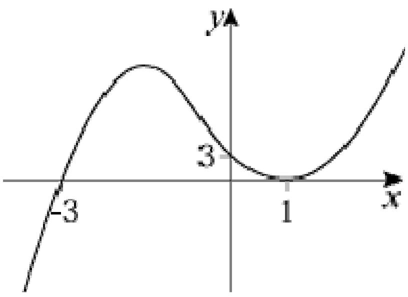
$x \to -\infty , y \to -\infty$



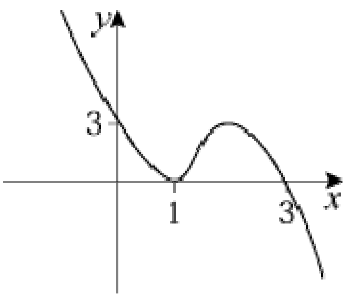
(f) $y = 0 \quad \Rightarrow \quad x = 0, 1$ (twice)
Turning point at (1 , 0).
$x \rightarrow \infty, y \rightarrow \infty$
$x \rightarrow -\infty, y \rightarrow -\infty$



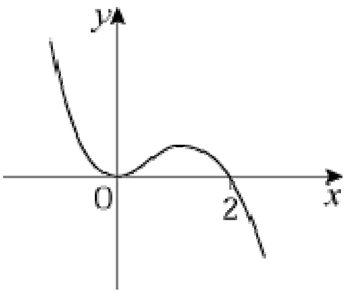
(g) $y = 0 \quad \Rightarrow \quad x = 1$ (twice), -3
$x = 0 \quad \Rightarrow \quad y = 3$
Turning point at (1 , 0).
$x \rightarrow \infty, y \rightarrow \infty$
$x \rightarrow -\infty, y \rightarrow -\infty$



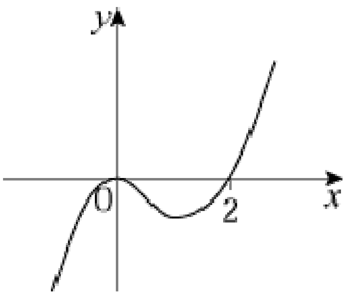
(h) $y = 0 \quad \Rightarrow \quad x = 1$ (twice), 3
$x = 0 \quad \Rightarrow \quad y = 3$
Turning point at (1 , 0).
$x \rightarrow \infty, y \rightarrow -\infty$
$x \rightarrow -\infty, y \rightarrow \infty$



(i) $y = 0 \quad \Rightarrow \quad x = 0$ (twice), 2
Turning point at (0 , 0).
$x \rightarrow \infty, y \rightarrow -\infty$
$x \rightarrow -\infty, y \rightarrow \infty$



(j) $y = 0 \quad \Rightarrow \quad x = 0$ (twice), 2
Turning point at (0 , 0).
$x \rightarrow \infty$, $y \rightarrow \infty$
$x \rightarrow -\infty$, $y \rightarrow -\infty$



Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sketching curves
Exercise A, Question 3

Question:

Factorise the following equations and then sketch the curves:

(a) $y = x^3 + x^2 - 2x$

(b) $y = x^3 + 5x^2 + 4x$

(c) $y = x^3 + 2x^2 + x$

(d) $y = 3x + 2x^2 - x^3$

(e) $y = x^3 - x^2$

(f) $y = x - x^3$

(g) $y = 12x^3 - 3x$

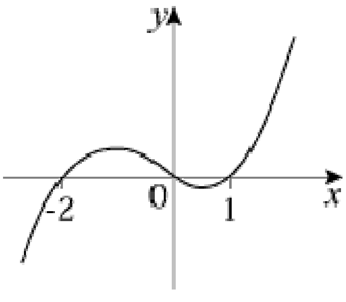
(h) $y = x^3 - x^2 - 2x$

(i) $y = x^3 - 9x$

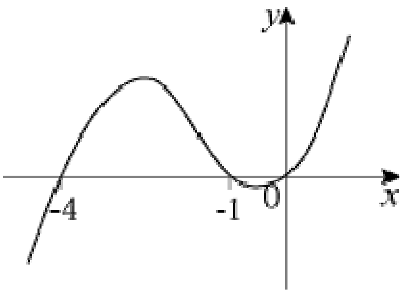
(j) $y = x^3 - 9x^2$

Solution:

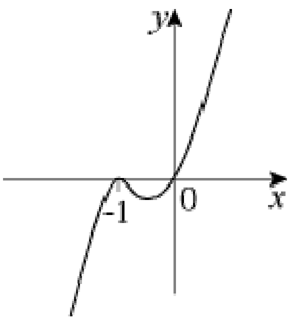
(a) $y = x^3 + x^2 - 2x = x(x^2 + x - 2)$
So $y = x(x + 2)(x - 1)$
$y = 0 \quad \Rightarrow \quad x = 0, 1, -2$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



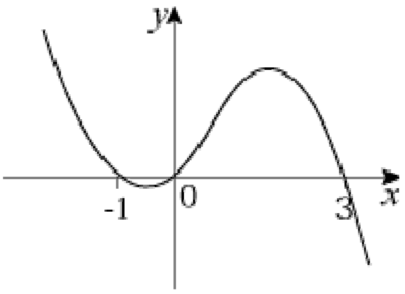
(b) $y = x^3 + 5x^2 + 4x = x(x^2 + 5x + 4)$
So $y = x(x + 4)(x + 1)$
$y = 0 \quad \Rightarrow \quad x = 0, -4, -1$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



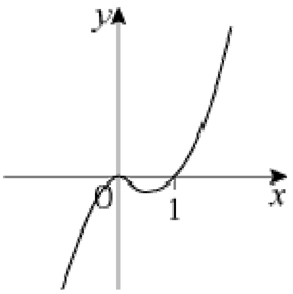
(c) $y = x^3 + 2x^2 + x = x(x^2 + 2x + 1)$
So $y = x(x + 1)^2$
$y = 0 \Rightarrow x = 0, -1$ (twice)
Turning point at $(-1, 0)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



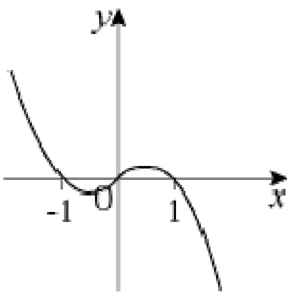
(d) $y = 3x + 2x^2 - x^3 = x(3 + 2x - x^2)$
So $y = x(3 - x)(1 + x)$
$y = 0 \Rightarrow x = 0, 3, -1$
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



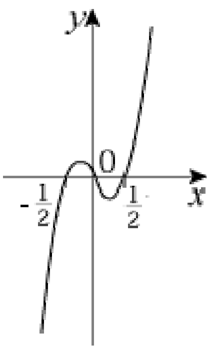
(e) $y = x^3 - x^2 = x^2(x - 1)$
$y = 0 \Rightarrow x = 0$ (twice), 1
Turning point at $(0, 0)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



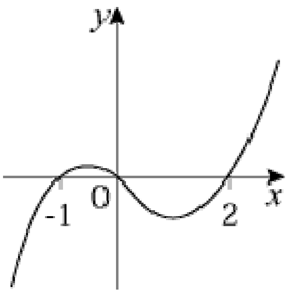
(f) $y = x - x^3 = x(1 - x^2)$
So $y = x(1 - x)(1 + x)$
$y = 0 \Rightarrow x = 0, 1, -1$
$x \rightarrow \infty, y \rightarrow -\infty$
$x \rightarrow -\infty, y \rightarrow \infty$



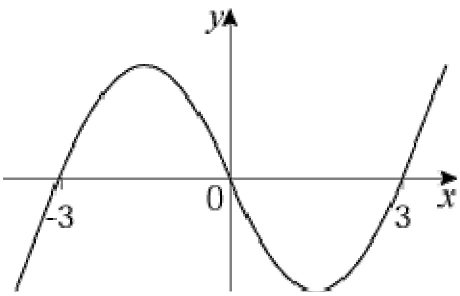
(g) $y = 12x^3 - 3x = 3x(4x^2 - 1)$
So $y = 3x(2x - 1)(2x + 1)$
$y = 0 \Rightarrow x = 0, \frac{1}{2}, -\frac{1}{2}$
$x \rightarrow \infty, y \rightarrow \infty$
$x \rightarrow -\infty, y \rightarrow -\infty$



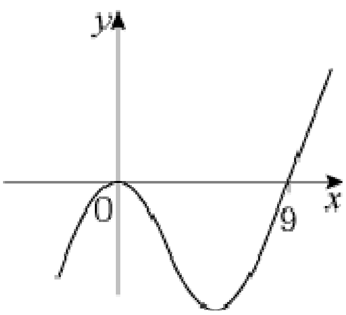
(h) $y = x^3 - x^2 - 2x = x(x^2 - x - 2)$
So $y = x(x + 1)(x - 2)$
$y = 0 \Rightarrow x = 0, -1, 2$
$x \rightarrow \infty, y \rightarrow \infty$
$x \rightarrow -\infty, y \rightarrow -\infty$



(i) $y = x^3 - 9x = x(x^2 - 9)$
So $y = x(x - 3)(x + 3)$
$y = 0 \Rightarrow x = 0, 3, -3$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



(j) $y = x^3 - 9x^2 = x^2(x - 9)$
$y = 0 \Rightarrow x = 0$ (twice), 9
Turning point at (0,0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sketching curves
Exercise B, Question 1

Question:

Sketch the following curves and show their positions relative to the curve $y = x^3$:

(a) $y = (x - 2)^3$

(b) $y = (2 - x)^3$

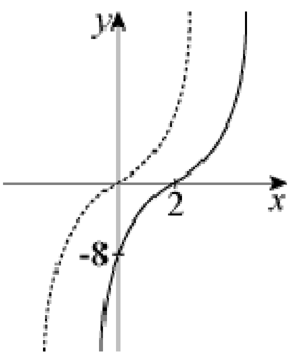
(c) $y = (x - 1)^3$

(d) $y = (x + 2)^3$

(e) $y = -(x + 2)^3$

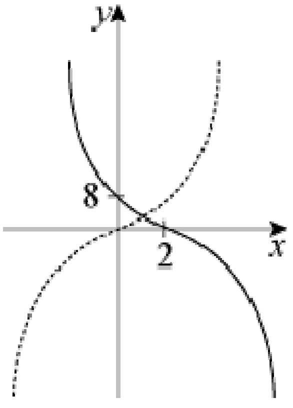
Solution:

(a) $y = 0 \Rightarrow x = 2$, so curve crosses x -axis at $(2, 0)$
$x = 0 \Rightarrow y = -8$, so curve crosses y-axis at $(0, -8)$



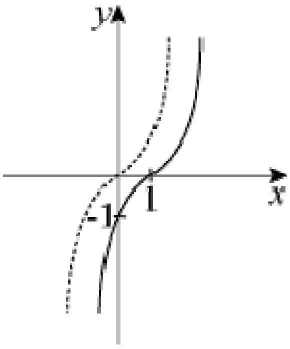
Curve is a translation of $+2$ in x direction of the curve $y = x^3$.

(b) $y = 0 \Rightarrow x = 2$, so curve crosses x-axis at $(2, 0)$
$x = 0 \Rightarrow y = 8$, so curve crosses y-axis at $(0, 8)$
$y = (2 - x)^3 = -(x - 2)^3$, so shape is like $y = -x^3$

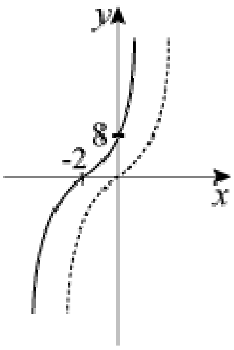


This is a horizontal translation of $+2$ of the curve $y = -x^3$.

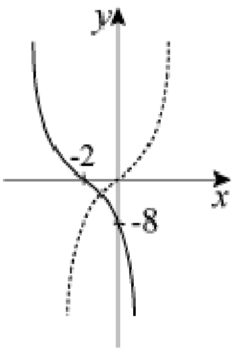
(c) $y = 0 \Rightarrow x = 1$, so curve crosses x-axis at $(1, 0)$
$x = 0 \Rightarrow y = -1$, so curve crosses y-axis at $(0, -1)$
$y = (x-1)^3$ is a horizontal translation of $+1$ of $y = x^3$.



(d) $y = 0 \Rightarrow x = -2$, so curve crosses x-axis at $(-2, 0)$
$x = 0 \Rightarrow y = 8$, so curve crosses y-axis at $(0, 8)$
$y = (x+2)^3$ is same shape as $y = x^3$ but translated horizontally by -2.



(e) $y = 0 \Rightarrow x = -2$, so curve crosses x-axis at $(-2, 0)$
$x = 0 \Rightarrow y = -8$, so curve crosses y-axis at $(0, -8)$
$y = -(x+2)^3$ is a reflection in x-axis of $y = -(x+2)^3$.



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Sketching curves
Exercise B, Question 2

Question:

Sketch the following and indicate the coordinates of the points where the curves cross the axes:

(a) $y = (x + 3)^3$

(b) $y = (x - 3)^3$

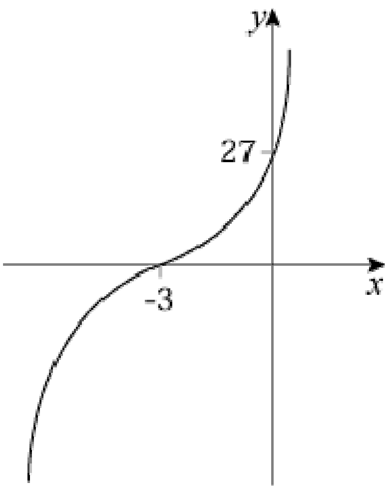
(c) $y = (1 - x)^3$

(d) $y = -(x - 2)^3$

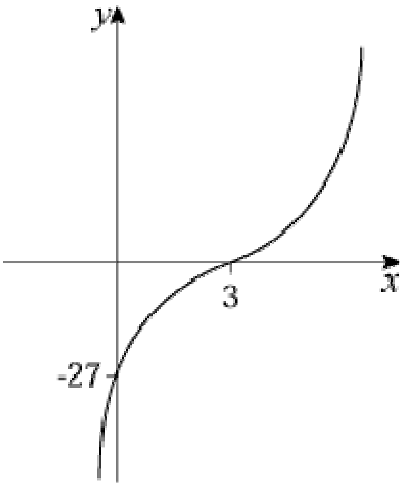
(e) $y = -(x - \frac{1}{2})^3$

Solution:

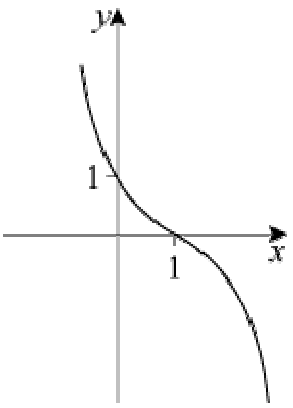
(a) $y = 0 \Rightarrow x = -3$, so curve crosses x-axis at $(-3, 0)$
$x = 0 \Rightarrow y = 27$, so curve crosses y-axis at $(0, 27)$
$y = (x + 3)^3$ is a translation of -3 in x-direction of $y = x^3$.



(b) $y = 0 \Rightarrow x = 3$, so curve crosses x-axis at $(3, 0)$
$x = 0 \Rightarrow y = -27$, so curve crosses y-axis at $(0, -27)$
$y = (x - 3)^3$ is a horizontal translation of $+3$ of $y = x^3$.

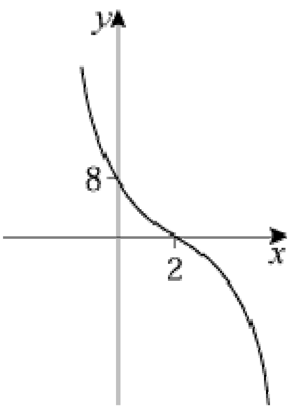


(c) $y = 0 \Rightarrow x = 1$, so curve crosses x-axis at $(1 , 0)$
$x = 0 \Rightarrow y = 1$, so curve crosses y-axis at $(0 , 1)$
$y = (1 - x)^3$ is a horizontal translation of $y = -x^3$.



Horizontal translation $+1$ of $y = -x^3$.

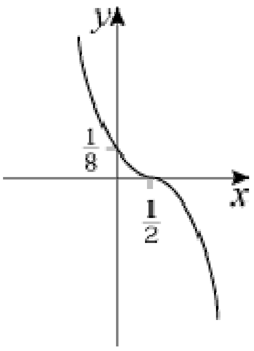
(d) $y = 0 \Rightarrow x = 2$, so curve crosses x-axis at $(2 , 0)$
$x = 0 \Rightarrow y = 8$, so curve crosses y-axis at $(0 , 8)$
$y = -(x - 2)^3$ is a translation ($+2$ in x-direction) of $y = -x^3$.



(e) $y = 0 \Rightarrow x = \frac{1}{2}$, so curve crosses x-axis at $(\frac{1}{2} , 0)$

$x = 0 \quad \Rightarrow \quad y = \frac{1}{8}$, so curve crosses y-axis at $(0\ ,\ \frac{1}{8})$

$y = -(x - \frac{1}{2})^3$ is a horizontal translation $(+\ \frac{1}{2})$ of $y = -x^3$.



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Sketching curves
Exercise C, Question 1

Question:

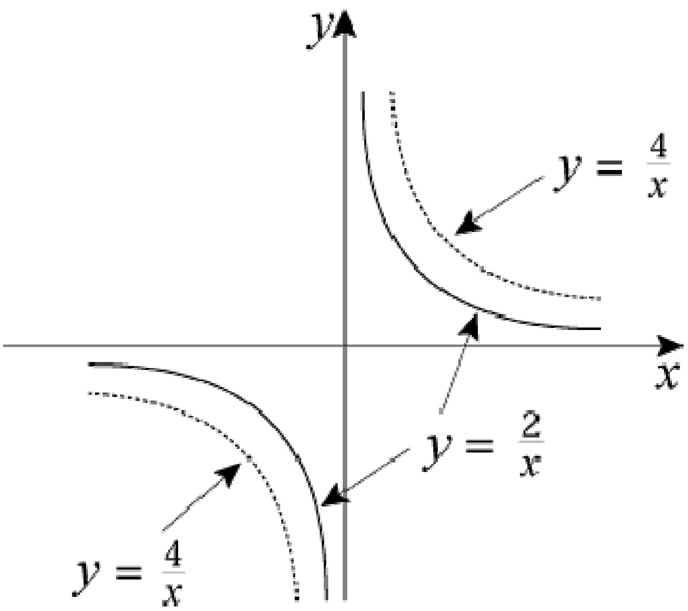
Sketch on the same diagram

$y = \frac{2}{x}$ and $y = \frac{4}{x}$

Solution:

For $x > 0$, $\frac{4}{x} > \frac{2}{x}$ (since $4 > 2$)

So $\frac{4}{x}$ is 'on top' of $\frac{2}{x}$ in 1st quadrant and 'below' in 3rd quadrant



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Sketching curves
Exercise C, Question 2

Question:

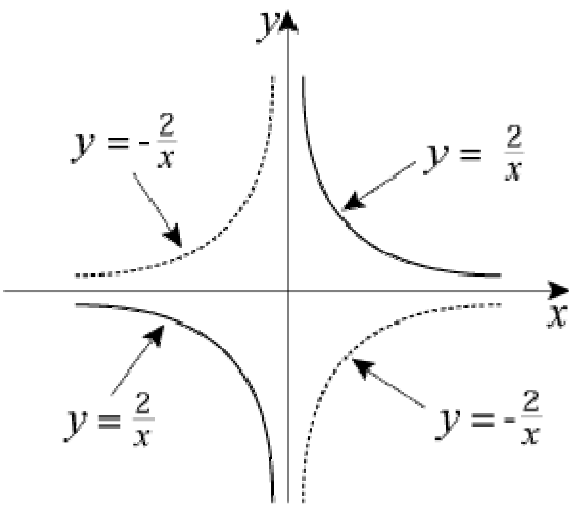
Sketch on the same diagram

$y = \frac{2}{x}$ and $y = -\frac{2}{x}$

Solution:

$y = \frac{2}{x} > 0$ for $x > 0$

$y = -\frac{2}{x} < 0$ for $x > 0$



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Sketching curves

Exercise C, Question 3

Question:

Sketch on the same diagram

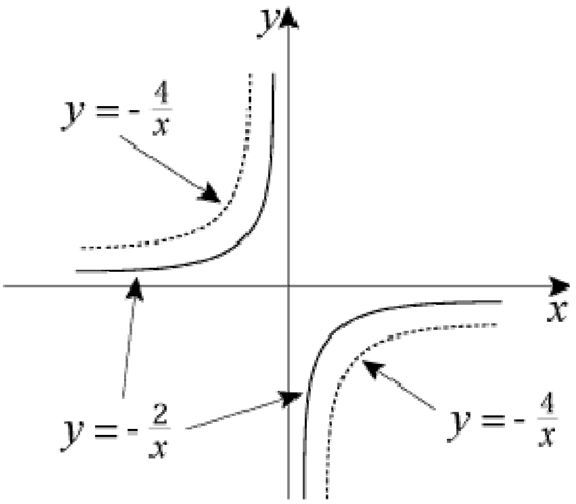
$y = -\frac{4}{x}$ and $y = -\frac{2}{x}$

Solution:

Graphs are like $y = -\frac{1}{x}$ and so exist in 2nd and 4th quadrants.

For $x < 0$, $-\frac{4}{x} > -\frac{2}{x}$

So $-\frac{4}{x}$ is ‘on top’ of $-\frac{2}{x}$ in 2nd quadrant and ‘below’ in 4th quadrant.



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Sketching curves
Exercise C, Question 4

Question:

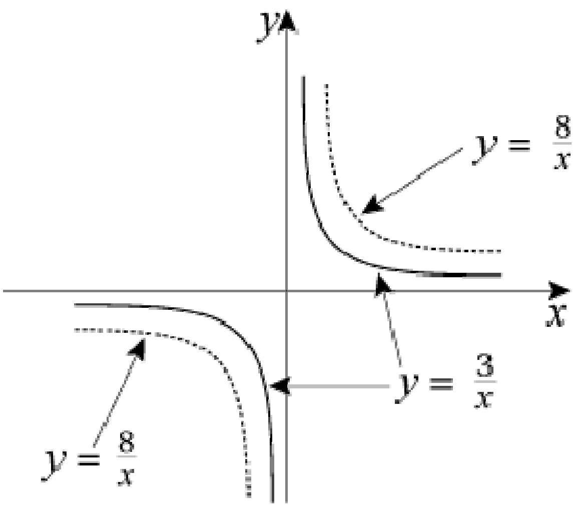
Sketch on the same diagram

$y = \frac{3}{x}$ and $y = \frac{8}{x}$

Solution:

For $x > 0$, $\frac{8}{x} > \frac{3}{x}$

So $y = \frac{8}{x}$ is ‘on top’ of $y = \frac{3}{x}$ in 1st quadrant and ‘below’ in 3rd quadrant.



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Sketching curves
Exercise C, Question 5

Question:

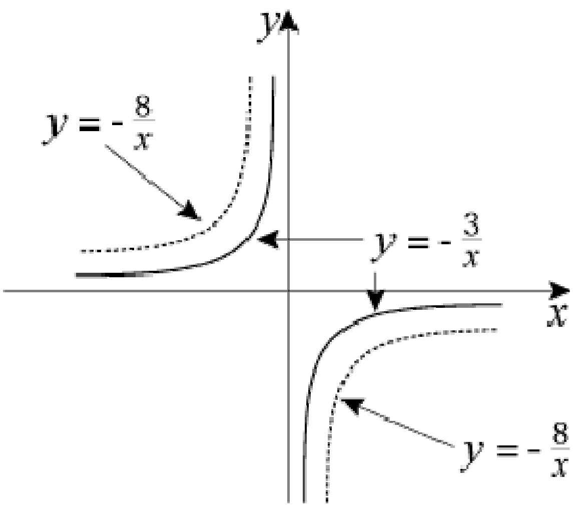
Sketch on the same diagram

$y = -\frac{3}{x}$ and $y = -\frac{8}{x}$

Solution:

For $x < 0$, $-\frac{8}{x} > -\frac{3}{x}$

So $y = -\frac{8}{x}$ is 'on top' of $y = -\frac{3}{x}$ in 2nd quadrant and 'below' in 4th quadrant.



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Sketching curves
Exercise D, Question 1

Question:

In each case:

(i) sketch the two curves on the same axes

(ii) state the number of points of intersection

(iii) write down a suitable equation which would give the x-coordinates of these points. (You are not required to solve this equation.)

(a) $y = x^2$, $y = x(x^2 - 1)$

(b) $y = x(x + 2)$, $y = -\frac{3}{x}$

(c) $y = x^2$, $y = (x + 1)(x - 1)^2$

(d) $y = x^2(1 - x)$, $y = -\frac{2}{x}$

(e) $y = x(x - 4)$, $y = \frac{1}{x}$

(f) $y = x(x - 4)$, $y = -\frac{1}{x}$

(g) $y = x(x - 4)$, $y = (x - 2)^3$

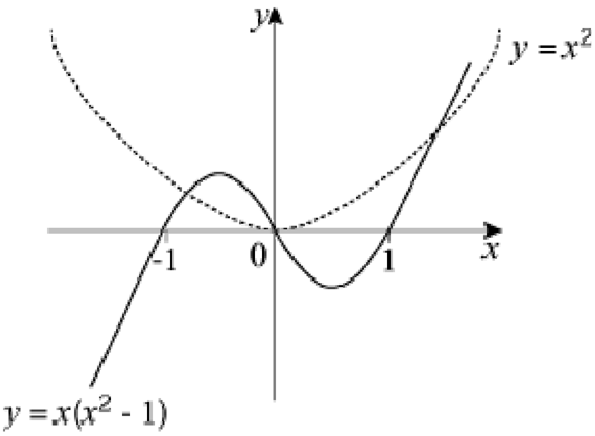
(h) $y = -x^3$, $y = -\frac{2}{x}$

(i) $y = -x^3$, $y = x^2$

(j) $y = -x^3$, $y = -x(x + 2)$

Solution:

(a) (i) $y = x^2$ is standard
$y = x(x^2 - 1) = x(x - 1)(x + 1)$
$y = 0 \quad \Rightarrow \quad x = 0, 1, -1$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



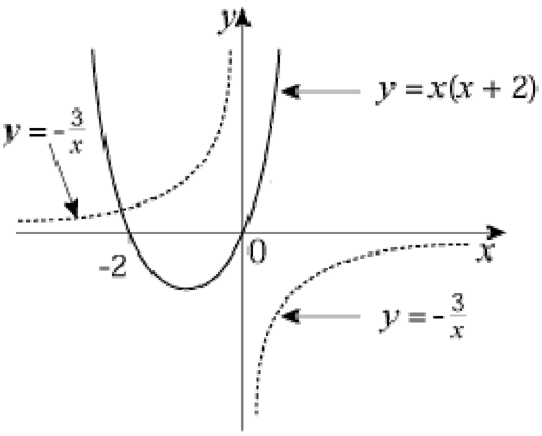
(ii) $y = x^2$ cuts $y = x(x^2 - 1)$ in 3 places.

(iii) Solutions given by $x^2 = x(x^2 - 1)$

(b) (i) $y = x(x + 2)$ is a $\cup$ -shaped curve

$y = 0 \Rightarrow x = 0, -2$

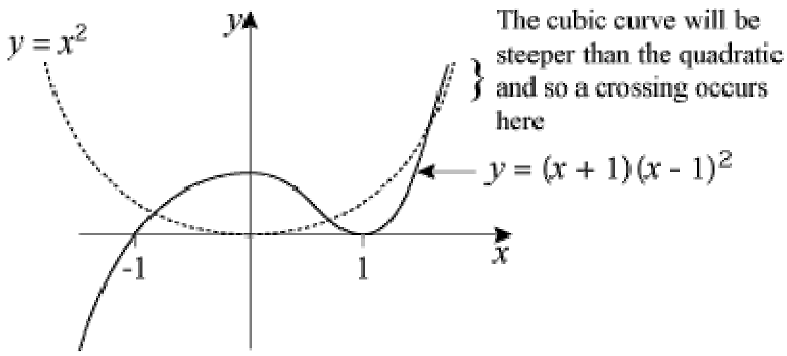
$y = -\frac{3}{x}$ is like $y = -\frac{1}{x}$



(ii) Curves cross at only 1 point.

(iii) Equation: $-\frac{3}{x} = x(x + 2)$

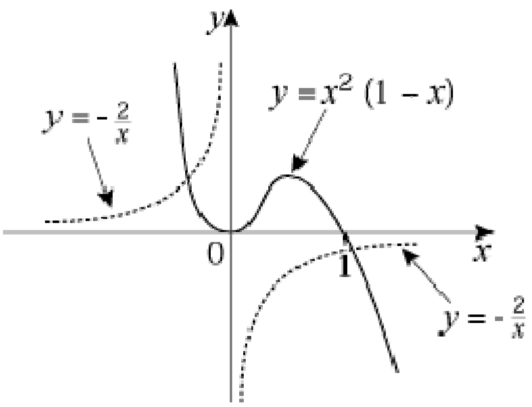
(c) (i) $y = x^2$ is standard
$y = (x + 1)(x - 1)^2$
$y = 0 \Rightarrow x = -1, 1$ (twice)
Turning point at (1 , 0)
$x \to \infty, y \to +\infty$
$x \to -\infty, y \to -\infty$



(ii) 3 points of intersection

(iii) Equation: $x^2 = (x + 1)(x - 1)^2$

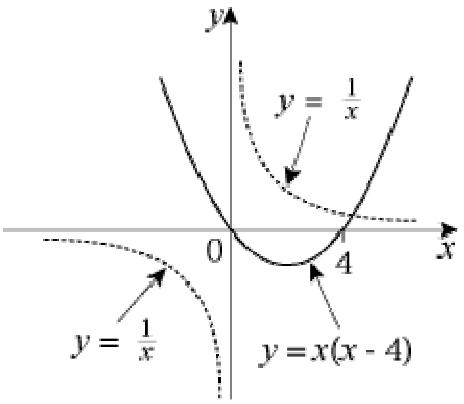
(d) (i) $y = x^2(1 - x)$
$y = 0 \quad \Rightarrow \quad x = 0$ (twice), 1
Turning point at (0 , 0)
$x \to \infty , y \to -\infty$
$x \to -\infty , y \to \infty$
$y = -\frac{2}{x}$ is like $y = -\frac{1}{x}$ and in 2nd and 4th quadrants



(ii) 2 points of intersection

(iii) Equation: $-\frac{2}{x} = x^2(1 - x)$

(e) (i) $y = x(x - 4)$ is a $\cup$ -shaped curve
$y = 0 \quad \Rightarrow \quad x = 0, 4$
$y = \frac{1}{x}$ is standard



(ii) 1 point of intersection

(iii) Equation: $\frac{1}{x} = x(x - 4)$

(f) (i) $y = x(x - 4)$ is a ∪ -shaped curve

$y = 0 \Rightarrow x = 0, 4$

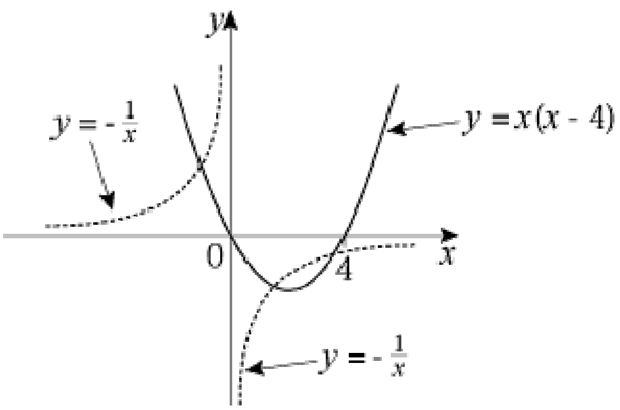
$y = -\frac{1}{x}$ is standard and in 2nd and 4th quadrants

At $x = 2$,

$y = -\frac{1}{x}$ gives $y = -\frac{1}{2}$

$y = x(x - 4)$ gives $y = 2(-2) = -4$

So $y = -\frac{1}{x}$ cuts $y = x(x - 4)$ in 4th quadrant.



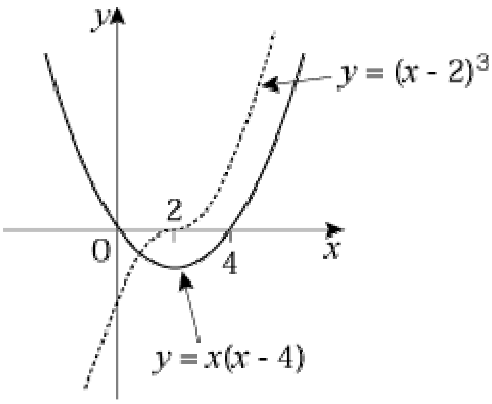
(ii) 3 points of intersection

(iii) Equation: $-\frac{1}{x} = x(x - 4)$

(g) (i) $y = x(x - 4)$ is a ∪ -shaped curve

$y = 0 \Rightarrow x = 0, 4$

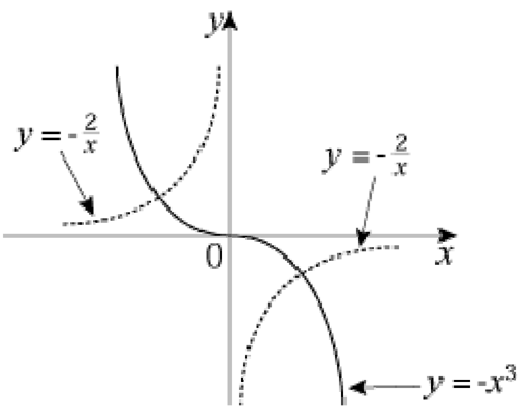
$y = (x - 2)^3$ is a translation of $+2$ in the x-direction of $y = x^3$.



(ii) 1 point of intersection

(iii) $x(x-4)=(x-2)^3$

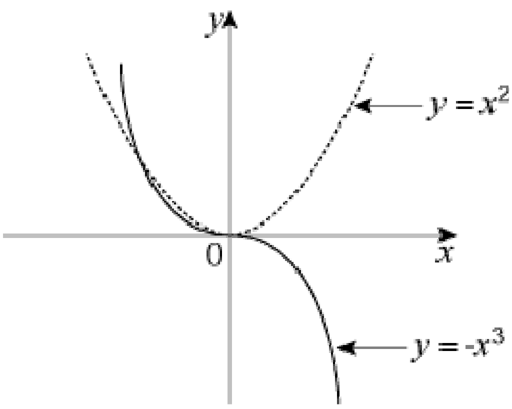
(h) (i) $y=-x^3$ is standard
$y=-\frac{2}{x}$ is like $y=-\frac{1}{x}$ and in 2nd and 4th quadrants.



(ii) 2 points of intersection

(iii) $-x^3=-\frac{2}{x}$ or $x^3=\frac{2}{x}$

(i) (i) $y=-x^3$ is standard
$y=x^2$ is standard

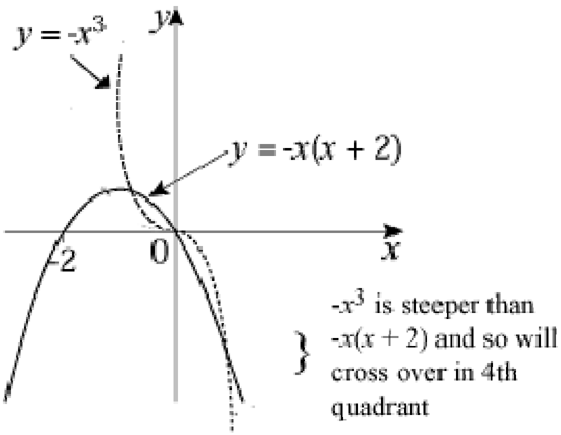


(ii) 2 points of intersection

[At (0,0) the curves actually touch. They intersect in the second quadrant.]

(iii) $x^2 = -x^3$

(j) (i) $y = -x^3$ is standard
$y = -x(x + 2)$ is ∩ shaped
$y = 0 \Rightarrow x = 0, -2$



(ii) 3 points of intersection

(iii) $-x^3 = -x(x + 2)$ or $x^3 = x(x + 2)$

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Sketching curves
Exercise D, Question 2

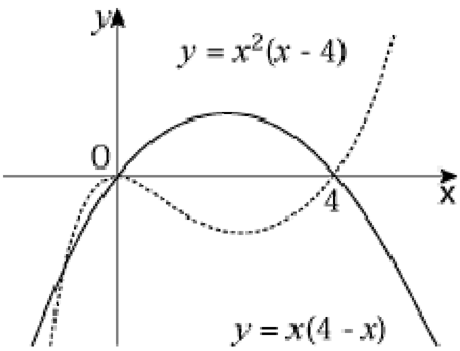
Question:

(a) On the same axes sketch the curves given by $y = x^2(x - 4)$ and $y = x(4 - x)$.

(b) Find the coordinates of the points of intersection.

Solution:

(a) $y = x^2(x - 4)$
$y = 0 \quad \Rightarrow \quad x = 0$ (twice), 4
Turning point at (0 , 0)
$y = x(4 - x)$ is $\cap$ shaped
$y = 0 \quad \Rightarrow \quad x = 0, 4$



(b) $x(4 - x) = x^2(x - 4)$
$\Rightarrow \quad 0 = x^2(x - 4) - x(4 - x)$
Factorise: $0 = x(x - 4)(x + 1)$
So intersections at $x = 0, -1, 4$
So points are [using $y = x(4 - x)$] (0, 0) ; (− 1, − 5) ; (4, 0)

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Sketching curves
Exercise D, Question 3

Question:

(a) On the same axes sketch the curves given by $y = x(2x + 5)$ and $y = x(1 + x)^2$

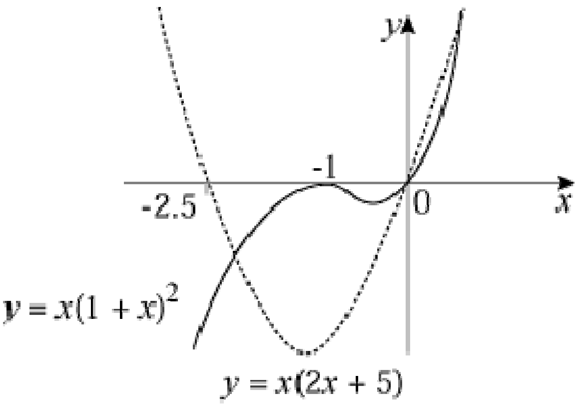
(b) Find the coordinates of the points of intersection.

Solution:

(a) $y = x(2x + 5)$ is $\cup$ shaped

$y = 0 \Rightarrow x = 0, -\frac{5}{2}$

$y = x(1 + x)^2$
$y = 0 \Rightarrow x = 0, -1$ (twice)
Turning point at $(-1, 0)$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



(b) $x(1 + x)^2 = x(2x + 5)$
$\Rightarrow x[x^2 + 2x + 1 - (2x + 5)] = 0$
$\Rightarrow x(x^2 - 4) = 0$
$\Rightarrow x(x - 2)(x + 2) = 0$
$\Rightarrow x = 0, 2, -2$
So points are [using $y = x(2x + 5)$]: (0, 0) ; (2, 18) ; (− 2, − 2)

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Sketching curves
Exercise D, Question 4

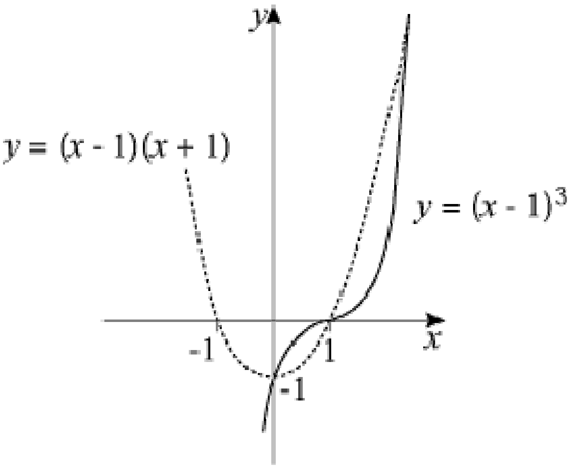
Question:

(a) On the same axes sketch the curves given by $y = (x - 1)^3$ and $y = (x - 1)(1 + x)$.

(b) Find the coordinates of the points of intersection.

Solution:

(a) $y = (x - 1)^3$ is like $y = x^3$ with crossing points at $(1, 0)$ and $(0, -1)$
$y = (x - 1)(1 + x)$ is a $\cup$ -shaped curve.
$y = 0 \Rightarrow x = 1, -1$



(b) Intersect when $(x - 1)^3 = (x - 1)(x + 1)$
i.e. $(x - 1)^3 - (x - 1)(x + 1) = 0$
$\Rightarrow (x - 1)[x^2 - 2x + 1 - (x + 1)] = 0$
$\Rightarrow (x - 1)(x^2 - 3x) = 0$
$\Rightarrow (x - 1)(x - 3)x = 0$
$\Rightarrow x = 0, 1, 3$
So intersections at $(0, -1)$; $(1, 0)$; $(3, 8)$

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Sketching curves
Exercise D, Question 5

Question:

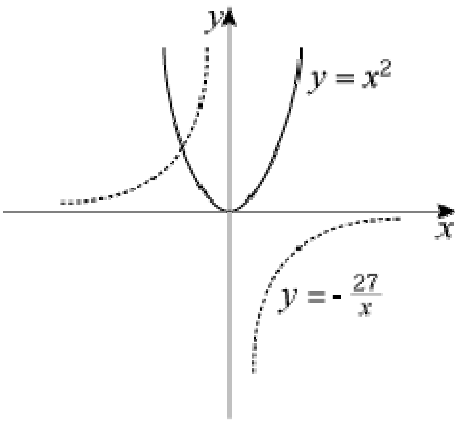
(a) On the same axes sketch the curves given by $y = x^2$ and $y = -\frac{27}{x}$.

(b) Find the coordinates of the point of intersection.

Solution:

(a) $y = -\frac{27}{x}$ is like $y = -\frac{1}{x}$ and in 2nd and 4th quadrants.

$y = x^2$ is standard



(b) $-\frac{27}{x} = x^2$

$\Rightarrow \quad -27 = x^3$

$\Rightarrow \quad x = -3$

Substitute in $y = -\frac{27}{x} \quad \Rightarrow \quad y = 9$

So intersection at $(-3, 9)$

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Sketching curves
Exercise D, Question 6

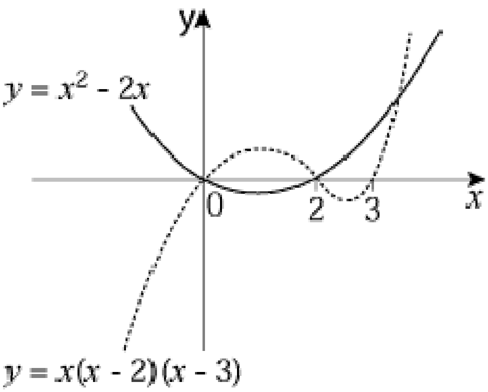
Question:

(a) On the same axes sketch the curves given by $y = x^2 - 2x$ and $y = x(x - 2)(x - 3)$.

(b) Find the coordinates of the point of intersection.

Solution:

(a) $y = x(x - 2)(x - 3)$
$y = 0 \quad \Rightarrow \quad x = 0, 2, 3$
$y = x^2 - 2x = x(x - 2)$ is $\cup$ shaped
$y = 0 \quad \Rightarrow \quad x = 0, 2$



(b) $x(x - 2) = x(x - 2)(x - 3)$
$\Rightarrow \quad 0 = x(x - 2)(x - 3 - 1)$
$\Rightarrow \quad 0 = x(x - 2)(x - 4)$
$\Rightarrow \quad x = 0, 2, 4$
Substitute in $y = x(x - 2) \quad \Rightarrow \quad y = 0, 0, 8$
So intersections at (0 , 0); (2 , 0); (4 , 8)

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Sketching curves
Exercise D, Question 7

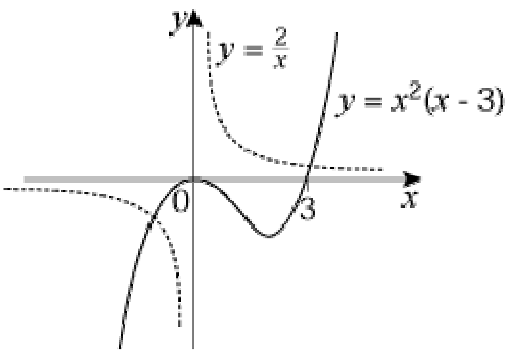
Question:

(a) On the same axes sketch the curves given by $y = x^2(x - 3)$ and $y = \frac{2}{x}$.

(b) Explain how your sketch shows that there are only two solutions to the equation $x^3(x - 3) = 2$.

Solution:

(a) $y = x^2(x - 3)$
$y = 0 \quad \Rightarrow \quad x = 0$ (twice), 3
Turning point at (0 , 0)
$y = \frac{2}{x}$ is like $y = \frac{1}{x}$



(b) Curves only cross at two points. So two solutions to
$\frac{2}{x} = x^2(x - 3)$
$2 = x^3(x - 3)$

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Sketching curves
Exercise D, Question 8

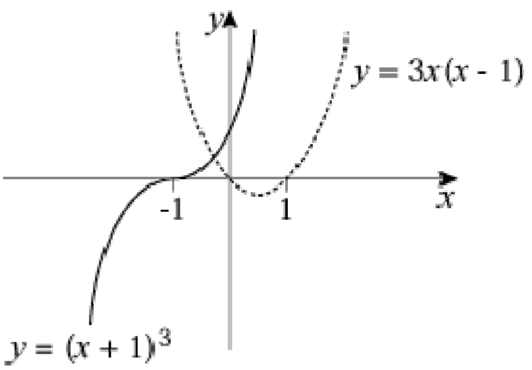
Question:

(a) On the same axes sketch the curves given by $y = (x + 1)^3$ and $y = 3x(x - 1)$.

(b) Explain how your sketch shows that there is only one solution to the equation $x^3 + 6x + 1 = 0$.

Solution:

(a) $y = (x + 1)^3$ is like $y = x^3$ and crosses at $(-1, 0)$ and $(0, 1)$.
$y = 3x(x - 1)$ is $\cup$ shaped
$y = 0 \Rightarrow x = 0, 1$



(b) Curves only cross once. So only one solution to
$(x + 1)^3 = 3x(x - 1)$
$x^3 + \cancel{3x^2} + 3x + 1 = \cancel{3x^2} - 3x$
$x^3 + 6x + 1 = 0$

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Sketching curves
Exercise D, Question 9

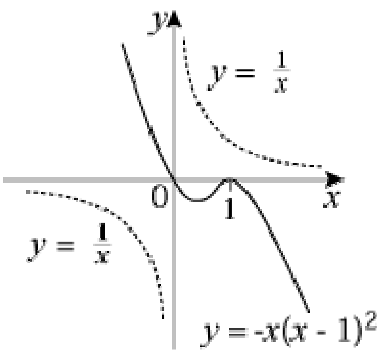
Question:

(a) On the same axes sketch the curves given by $y = \frac{1}{x}$ and $y = -x(x-1)^2$.

(b) Explain how your sketch shows that there are no solutions to the equation $1 + x^2(x-1)^2 = 0$.

Solution:

(a) $y = -x(x-1)^2$
$y = 0 \quad \Rightarrow \quad x = 0, 1$ (twice)
Turning point at (1 , 0)
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



(b) Curves do not cross, so no solutions to

$\frac{1}{x} = -x(x-1)^2$

$1 = -x^2(x-1)^2$
$1 + x^2(x-1)^2 = 0$

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Sketching curves
Exercise D, Question 10

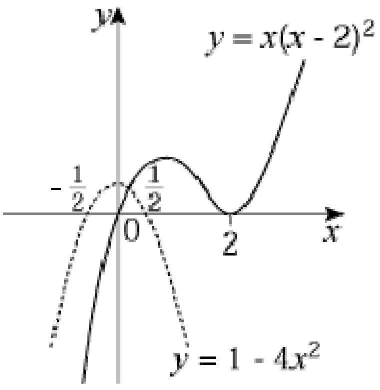
Question:

(a) On the same axes sketch the curves given by $y = 1 - 4x^2$ and $y = x(x - 2)^2$.

(b) State, with a reason, the number of solutions to the equation $x^3 + 4x - 1 = 0$.

Solution:

(a) $y = x(x - 2)^2$
$y = 0 \Rightarrow x = 0, 2$ (twice)
Turning point at (2 , 0)
$x \to \infty , y \to \infty$
$x \to -\infty , y \to -\infty$
$y = 1 - 4x^2 = (1 - 2x)(1 + 2x)$ is ∩ shaped
$y = 0 \Rightarrow x = \frac{1}{2}, -\frac{1}{2}$



(b) Curves cross once. So one solution to
$1 - 4x^2 = x(x - 2)^2$
$1 - 4x^2 = x(x^2 - 4x + 4)$
$1 - 4x^2 = x^3 - 4x^2 + 4x$
$0 = x^3 + 4x - 1$

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Sketching curves
Exercise D, Question 11

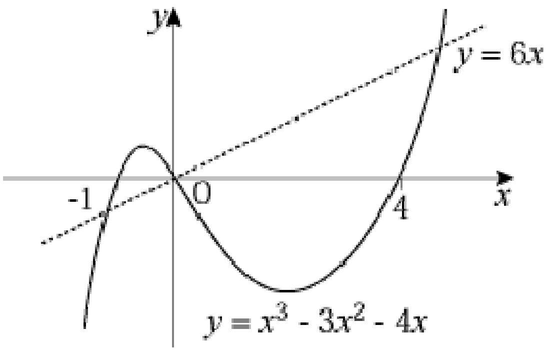
Question:

(a) On the same axes sketch the curve $y = x^3 - 3x^2 - 4x$ and the line $y = 6x$.

(b) Find the coordinates of the points of intersection.

Solution:

(a) $y = x^3 - 3x^2 - 4x = x(x^2 - 3x - 4)$
So $y = x(x - 4)(x + 1)$
$y = 0 \Rightarrow x = 0, -1, 4$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$
$y = 6x$ is a straight line through $(0, 0)$



(b) $x^3 - 3x^2 - 4x = 6x$
$\Rightarrow x^3 - 3x^2 - 10x = 0$
$\Rightarrow x(x^2 - 3x - 10) = 0$
$\Rightarrow x(x - 5)(x + 2) = 0$
$\Rightarrow x = 0, 5, -2$
So (using $y = 6x$) the points of intersection are: $(0, 0)$; $(5, 30)$; $(-2, -12)$

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Sketching curves
Exercise D, Question 12

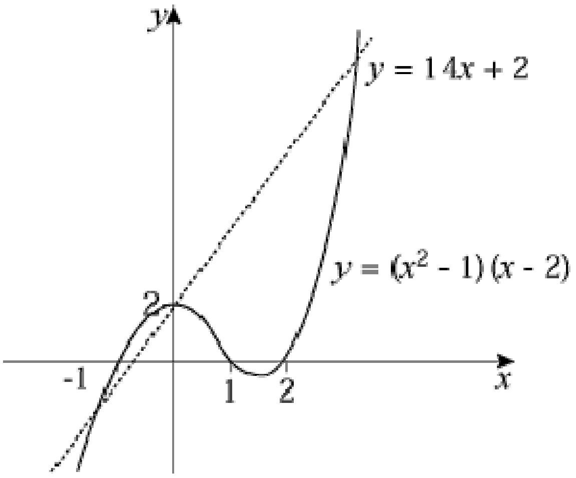
Question:

(a) On the same axes sketch the curve $y = (x^2 - 1)(x - 2)$ and the line $y = 14x + 2$.

(b) Find the coordinates of the points of intersection.

Solution:

(a) $y = (x^2 - 1)(x - 2) = (x - 1)(x + 1)(x - 2)$
$y = 0 \quad \Rightarrow \quad x = 1, \ -1, 2$
$x = 0 \quad \Rightarrow \quad y = 2$
$x \to \infty , y \to \infty$
$x \to -\infty , y \to -\infty$
$y = 14x + 2$ is a straight line passing through (0 , 2) and $(-\frac{1}{7} , 0)$.



(b) Intersection when $14x + 2 = (x^2 - 1)(x - 2)$
$\Rightarrow \quad 14x + 2 = x^3 - 2x^2 - x + 2$
$\Rightarrow \quad 0 = x^3 - 2x^2 - 15x$
$\Rightarrow \quad 0 = x(x^2 - 2x - 15)$
$\Rightarrow \quad 0 = x(x - 5)(x + 3)$
$\Rightarrow \quad x = 0, 5, -3$
So (using $y = 14x + 2$) the points of intersection are: (0 , 2); (5 , 72); (− 3 , − 40)

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Sketching curves
Exercise D, Question 13

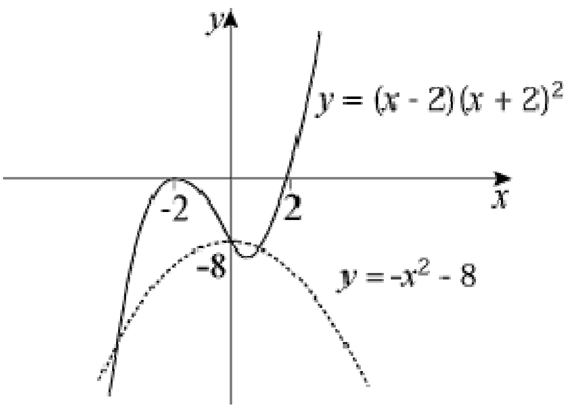
Question:

(a) On the same axes sketch the curves with equations $y = (x - 2)(x + 2)^2$ and $y = -x^2 - 8$.

(b) Find the coordinates of the points of intersection.

Solution:

(a) $y = (x - 2)(x + 2)^2$
$y = 0 \Rightarrow x = -2$ (twice), 2
$x = 0 \Rightarrow y = -8$
Turning point at $(-2, 0)$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$
$y = -x^2 - 8$ is $\cap$ shaped with a maximum at $(0, -8)$



(b) Intersections when $-x^2 - 8 = (x + 2)^2(x - 2)$
$\Rightarrow -x^2 - 8 = (x^2 + 4x + 4)(x - 2)$
$\Rightarrow -x^2 - 8 = x^3 + 4x^2 + 4x - 2x^2 - 8x - 8$
$\Rightarrow 0 = x^3 + 3x^2 - 4x$
$\Rightarrow 0 = x(x^2 + 3x - 4)$
$\Rightarrow 0 = x(x + 4)(x - 1)$
$\Rightarrow x = 0, 1, -4$
So (using $y = -x^2 - 8$) points of intersection are: $(0, -8)$; $(1, -9)$; $(-4, -24)$

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Sketching curves
Exercise E, Question 1

Question:

Apply the following transformations to the curves with equations y = f(x) where:

(i) f(x) = x^2

(ii) f(x) = x^3

(iii) f(x) = $\frac{1}{x}$

In each case state the coordinates of points where the curves cross the axes and in (iii) state the equations of any asymptotes.

(a) f(x + 2)

(b) f(x) + 2

(c) f(x − 1)

(d) f(x) − 1

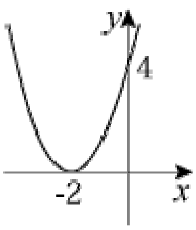
(e) f(x) − 3

(f) f(x − 3)

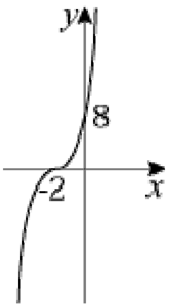
Solution:

(a) f(x + 2) is a horizontal translation of − 2.

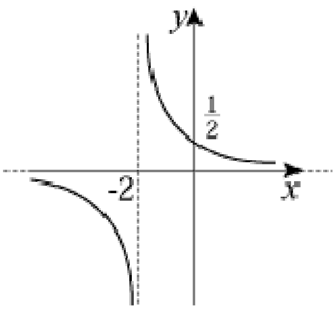
(i) $y = x^2 \rightarrow y = (x+2)^2$



(ii) $y = x^3 \rightarrow y = (x+2)^3$



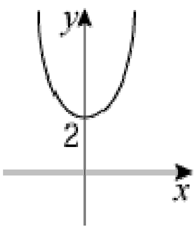
(iii) $y = \frac{1}{x} \to y = \frac{1}{x + 2}$



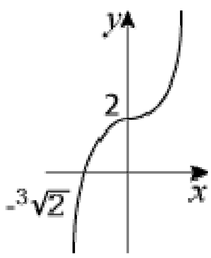
Asymptotes: $x = -2$ and $y = 0$

(b) f(x) + 2 is a vertical translation of + 2.

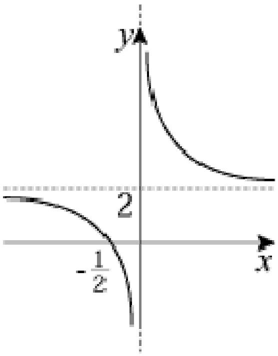
(i) $y = x^2 \to y = x^2 + 2$



(ii) $y = x^3 \to y = x^3 + 2$



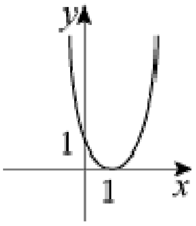
(iii) $y = \frac{1}{x} \to y = \frac{1}{x} + 2$



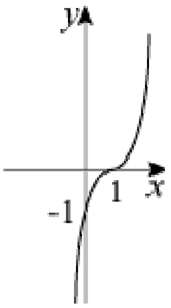
Asymptotes: $x = 0$ and $y = 2$

(c) $f(x - 1)$ is a horizontal translation of + 1.

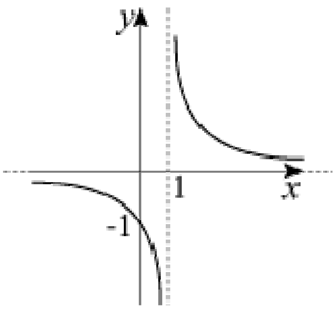
(i) $y = x^2 \rightarrow y = (x - 1)^2$



(ii) $y = x^3 \rightarrow y = (x - 1)^3$



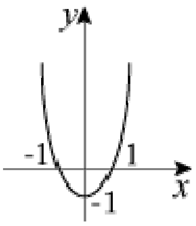
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{x - 1}$



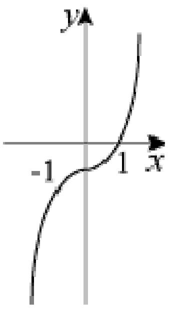
Asymptotes: $x = 1$, $y = 0$

(d) $f(x) - 1$ is a vertical translation of − 1.

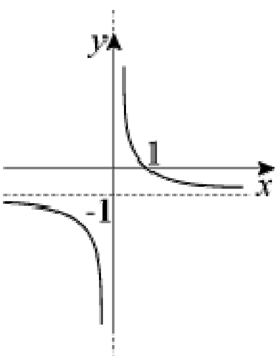
(i) $y = x^2 \rightarrow y = x^2 - 1$



(ii) $y = x^3 \rightarrow y = x^3 - 1$



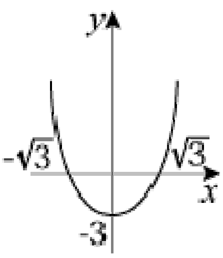
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{x} - 1$



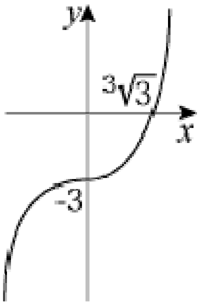
Asymptotes: $x = 0$, $y = -1$

(e) f(x) − 3 is a vertical translation of − 3.

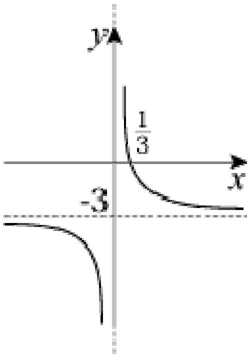
(i) $y = x^2 \rightarrow y = x^2 - 3$



(ii) $y = x^3 \rightarrow y = x^3 - 3$



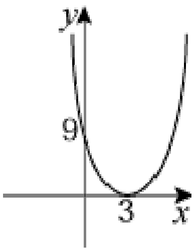
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{x} - 3$



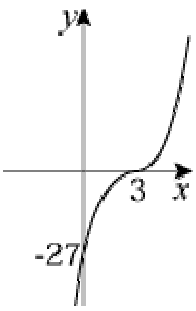
Asymptotes: $x = 0$, $y = -3$

(f) f$(x - 3)$ is a horizontal translation of $+3$.

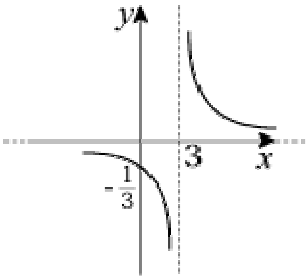
(i) $y = x^2 \rightarrow y = (x - 3)^2$



(ii) $y = x^3 \rightarrow y = (x - 3)^3$



(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{x - 3}$



Asymptotes: $x = 3$, $y = 0$

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Sketching curves
Exercise E, Question 2

Question:

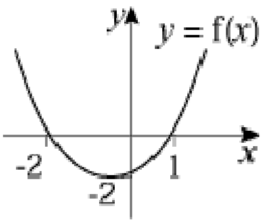
(a) Sketch the curve $y = f(x)$ where $f(x) = (x - 1)(x + 2)$.

(b) On separate diagrams sketch the graphs of (i) $y = f(x + 2)$ (ii) $y = f(x) + 2$.

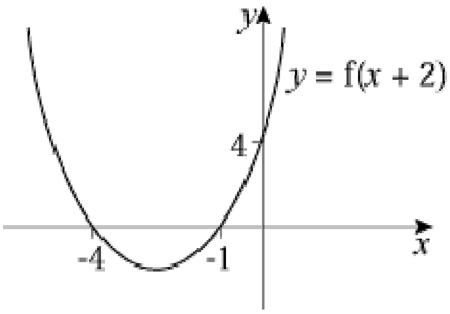
(c) Find the equations of the curves $y = f(x + 2)$ and $y = f(x) + 2$, in terms of x, and use these equations to find the coordinates of the points where your graphs in part (b) cross the y-axis.

Solution:

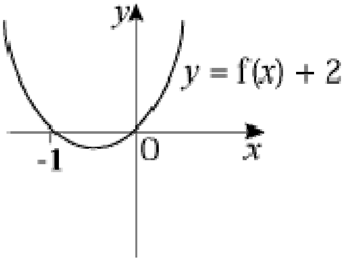
(a) $f(x) = 0 \quad \Rightarrow \quad x = 1, \ - 2$



(b)(i) $f(x + 2)$ is a horizontal translation of -2.



(ii) $f(x) + 2$ is a vertical translation of $+2$



Since axis of symmetry of $f(x)$ is at $x = -\frac{1}{2}$, the same axis of symmetry applies to $f(x) + 2$. Since one root is at $x = 0$, the other must be symmetric at $x = -1$.

(c) $y = f(x + 2)$ is $y = (x + 1)(x + 4)$. So $x = 0 \quad \Rightarrow \quad y = 4$

$y = \text{f}(x) + 2$ is $y = x^2 + x = x(x + 1)$. So $x = 0 \quad \Rightarrow \quad y = 0$

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Sketching curves
Exercise E, Question 3

Question:

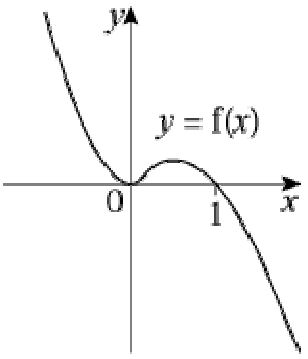
(a) Sketch the graph of $y = f(x)$ where $f(x) = x^2(1 - x)$.

(b) Sketch the curve with equation $y = f(x + 1)$.

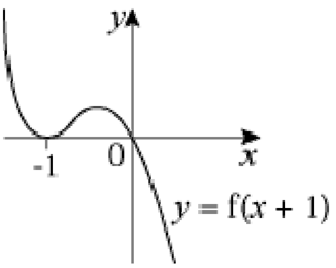
(c) By finding the equation $f(x + 1)$ in terms of x, find the coordinates of the point in part (b) where the curve crosses the y-axis.

Solution:

(a) $y = x^2(1 - x)$
$y = 0 \quad \Rightarrow \quad x = 0$ (twice), 1
Turning point at (0 , 0)
$x \to \infty , y \to - \infty$
$x \to - \infty , y \to \infty$



(b) $f(x + 1)$ is a horizontal translation of $- 1$.



(c) $f(x + 1) = (x + 1)^2 [1 - (x + 1)] = - (x + 1)^2 x$
So $y = 0 \quad \Rightarrow \quad x = 0$, i.e. curve passes through (0 , 0).

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Sketching curves
Exercise E, Question 4

Question:

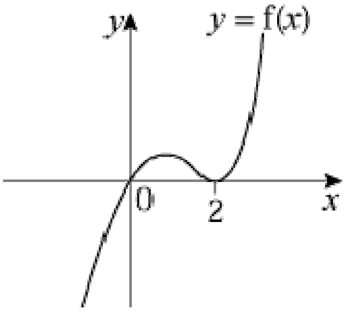
(a) Sketch the graph of $y = f(x)$ where $f(x) = x(x - 2)^2$.

(b) Sketch the curves with equations $y = f(x) + 2$ and $y = f(x + 2)$.

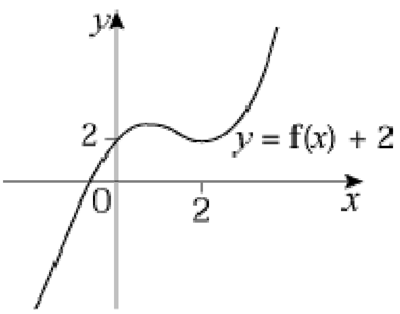
(c) Find the coordinates of the points where the graph of $y = f(x + 2)$ crosses the axes.

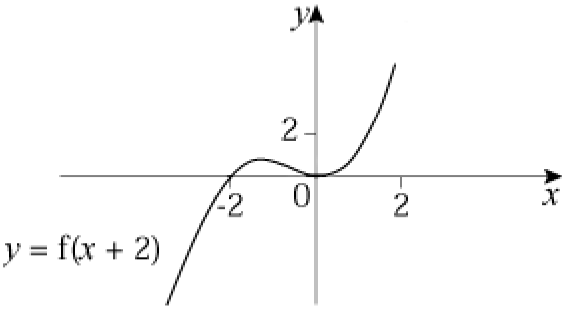
Solution:

(a) $y = x(x - 2)^2$
$y = 0 \quad \Rightarrow \quad x = 0, 2$ (twice)
Turning point at (2 , 0)
$x \rightarrow \infty , y \rightarrow \infty$
$x \rightarrow - \infty , y \rightarrow - \infty$



(b)





(c) $f(x + 2) = 0$ at points where $(x + 2) [(x + 2) - 2]^2 = 0$

⇒ $(x + 2)(x)^2 = 0$

⇒ $x = 0$ and $x = -2$

So graph crosses axes at $(0, 0)$; $(-2, 0)$.

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Sketching curves
Exercise E, Question 5

Question:

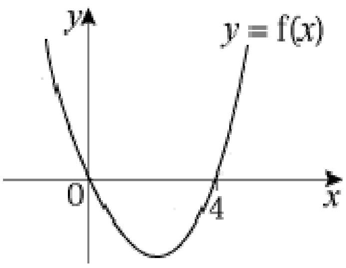
(a) Sketch the graph of $y = f(x)$ where $f(x) = x(x - 4)$.

(b) Sketch the curves with equations $y = f(x + 2)$ and $y = f(x) + 4$.

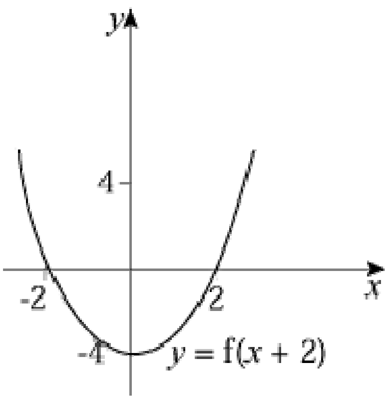
(c) Find the equations of the curves in part (b) in terms of x and hence find the coordinates of the points where the curves cross the axes.

Solution:

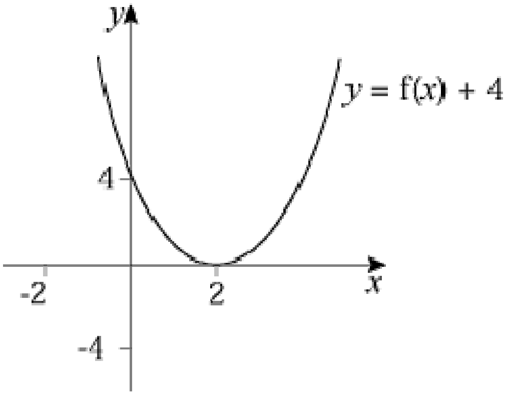
(a) $y = x(x - 4)$ is $\cup$ shaped and passes though (0 , 0) and (4 , 0).



(b) $f(x + 2)$ is a horizontal translation of -2.



$f(x) + 4$ is a vertical translation of $+4$.



(c) $f(x + 2) = (x + 2)[(x + 2) - 4] = (x + 2)(x - 2)$

$y = 0 \Rightarrow x = -2, 2$

$f(x) + 4 = x(x - 4) + 4 = x^2 - 4x + 4 = (x - 2)^2$

$y = 0 \Rightarrow x = 2$

The minimum point on $y = f(x)$ is when $x = 2$ (by symmetry) and then $f(2) = -4$.
So $y = f(x + 2)$ crosses y-axis at $(0, -4)$
and $y = f(x) + 4$ touches x-axis at $(2, 0)$.

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Sketching curves
Exercise F, Question 1

Question:

Apply the following transformations to the curves with equations $y = f(x)$ where:

(i) $f(x) = x^2$

(ii) $f(x) = x^3$

(iii) $f(x) = \frac{1}{x}$

In each case show both $f(x)$ and the transformation on the same diagram.

(a) $f(2x)$

(b) $f(-x)$

(c) $f(\frac{1}{2}x)$

(d) $f(4x)$

(e) $f(\frac{1}{4}x)$

(f) $2f(x)$

(g) $-f(x)$

(h) $4f(x)$

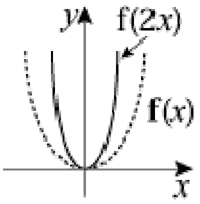
(i) $\frac{1}{2}f(x)$

(j) $\frac{1}{4}f(x)$

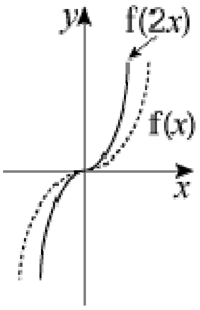
Solution:

(a) $f(2x)$ means multiply x-coordinates by $\frac{1}{2}$.

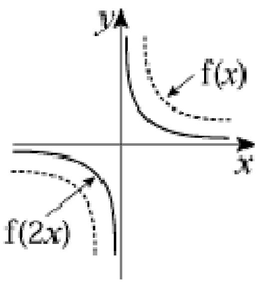
(i) $y = x^2 \rightarrow y = (2x)^2 = 4x^2$



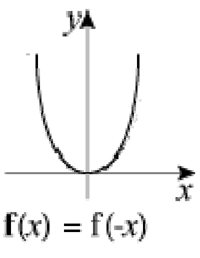
(ii) $y = x^3 \rightarrow y = (2x)^3 = 8x^3$



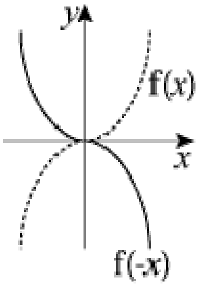
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{2x} = \frac{1}{2} \times \frac{1}{x}$



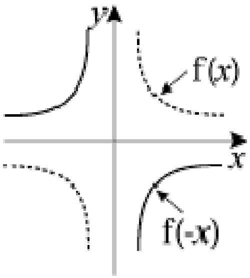
(b) (i) $y = x^2 \rightarrow y = (- x)^2 = x^2$



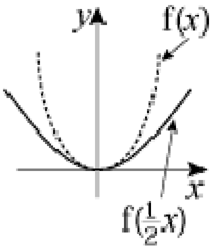
(ii) $y = x^3 \rightarrow y = (- x)^3 = - x^3$



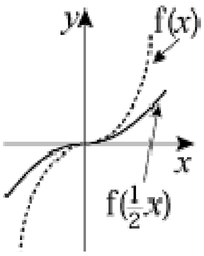
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{-x} = -\frac{1}{x}$



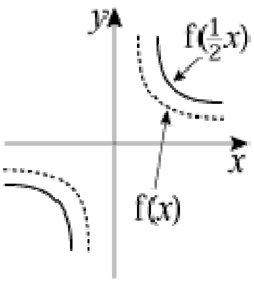
(c) (i) $y = x^3 \rightarrow y = (\frac{1}{2}x)^2 = \frac{x^2}{4}$



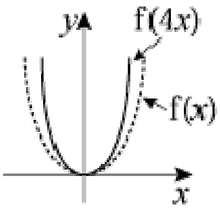
(ii) $y = x^3 \rightarrow y = (\frac{1}{2}x)^3 = \frac{x^3}{8}$



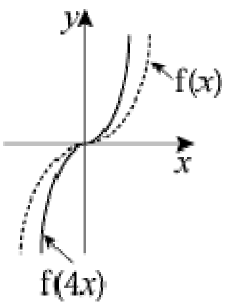
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{\frac{1}{2}x} = \frac{2}{x}$



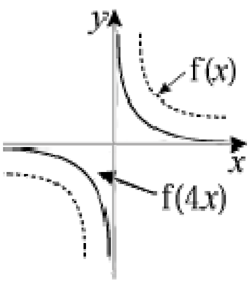
(d) (i) $y = x^2 \rightarrow y = (4x)^2 = 16x^2$



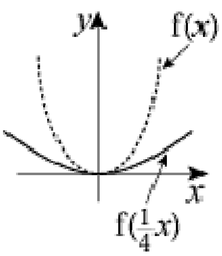
(ii) $y = x^3 \rightarrow y = (4x)^3 = 64x^3$



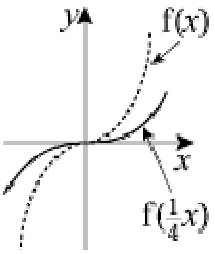
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{4x} = \frac{1}{4} \times \frac{1}{x}$



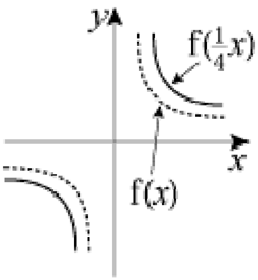
(e) (i) $y = x^2 \rightarrow y = (\frac{1}{4}x)^2 = \frac{x^2}{16}$



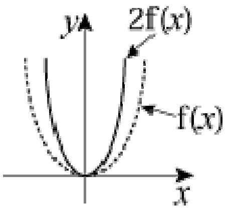
(ii) $y = x^3 \rightarrow y = (\frac{1}{4}x)^3 = \frac{x^3}{64}$



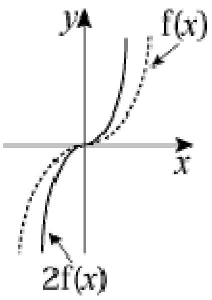
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{\frac{1}{4}x} = \frac{4}{x}$



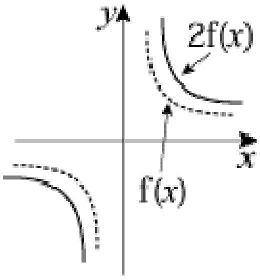
(f) (i) $y = x^2 \rightarrow y = 2x^2$



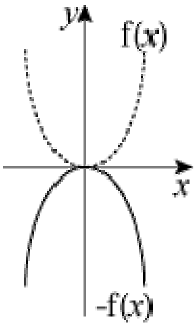
(ii) $y = x^3 \rightarrow y = 2x^3$



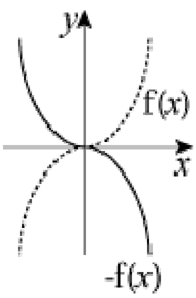
(iii) $y = \frac{1}{x} \rightarrow y = 2 \times \frac{1}{x} = \frac{2}{x}$



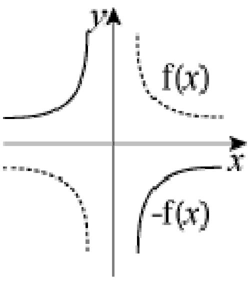
(g) (i) $y = x^2 \to y = -x^2$



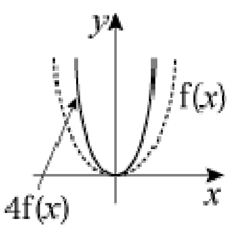
(ii) $y = x^3 \to y = -x^3$



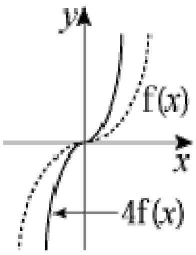
(iii) $y = \frac{1}{x} \to y = -\frac{1}{x}$



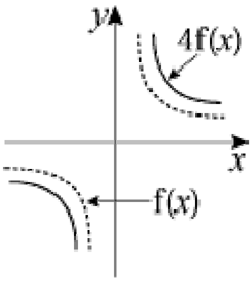
(h) (i) $y = x^2 \to y = 4x^2$



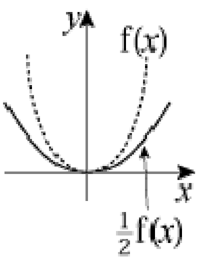
(ii) $y = x^3 \rightarrow y = 4x^3$



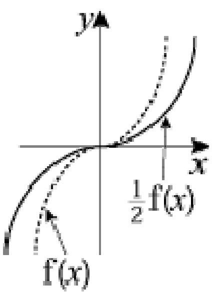
(iii) $y = \frac{1}{x} \rightarrow y = \frac{4}{x}$



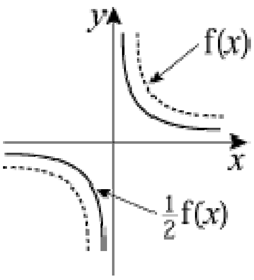
(i) (i) $y = x^2 \rightarrow y = \frac{1}{2}x^2$



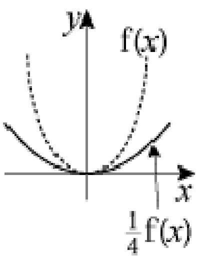
(ii) $y = x^3 \rightarrow y = \frac{1}{2}x^3$



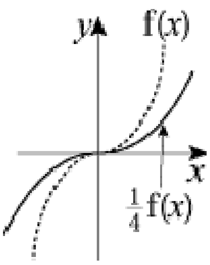
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{2} \times \frac{1}{x}$



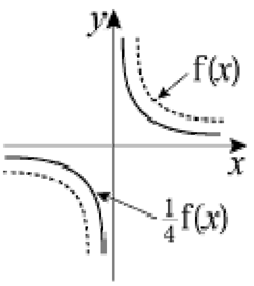
(j) (i) $y = x^2 \rightarrow y = \frac{1}{4}x^2$



(ii) $y = x^3 \rightarrow y = \frac{1}{4}x^3$



(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{4} \times \frac{1}{x}$



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Sketching curves
Exercise F, Question 2

Question:

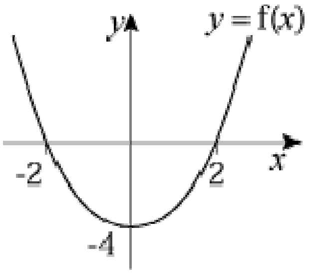
(a) Sketch the curve with equation $y = f(x)$ where $f(x) = x^2 - 4$.

(b) Sketch the graphs of $y = f(4x)$, $y = 3f(x)$, $y = f(-x)$ and $y = -f(x)$.

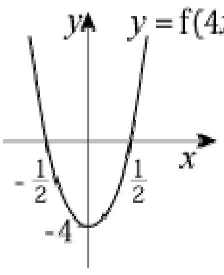
Solution:

(a) $y = x^2 - 4 = (x - 2)(x + 2)$ and is ∪ shaped

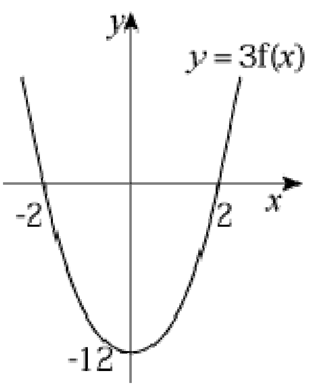
$y = 0 \Rightarrow x = 2, -2$



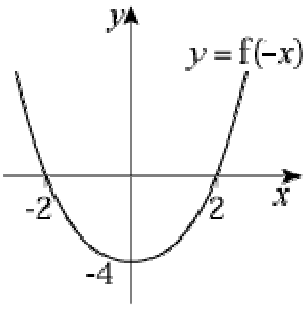
(b) $f(4x)$ is a stretch $\times \frac{1}{4}$ horizontally



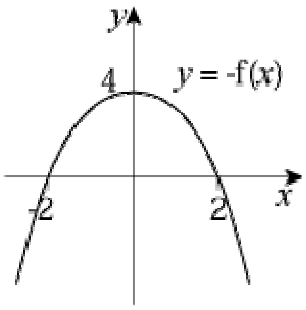
$3f(x)$ is a stretch ×3 vertically



$f(-x)$ is a reflection in y-axis



− f(x) is a reflection in x-axis



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Sketching curves
Exercise F, Question 3

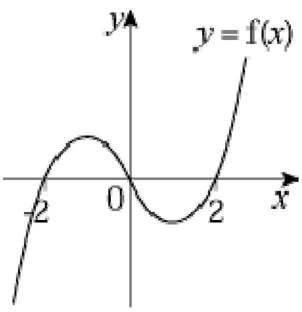
Question:

(a) Sketch the curve with equation $y = f(x)$ where $f(x) = (x - 2)(x + 2)x$.

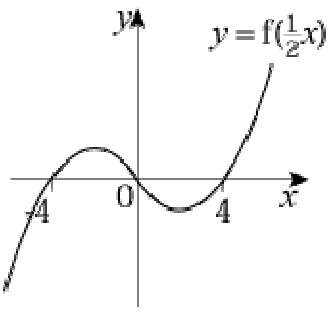
(b) Sketch the graphs of $y = f(\frac{1}{2}x)$, $y = f(2x)$ and $y = -f(x)$.

Solution:

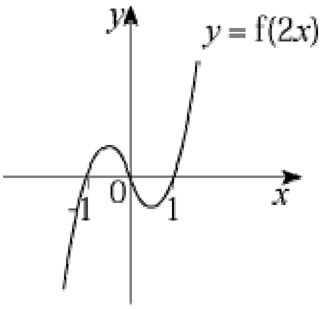
(a) $y = (x - 2)(x + 2)x$
$y = 0 \quad \Rightarrow \quad x = 2, -2, 0$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



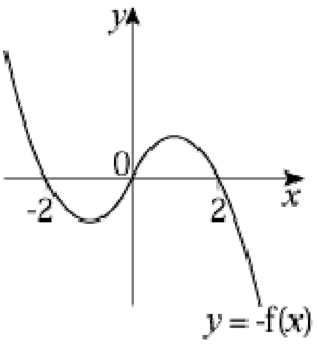
(b) $f(\frac{1}{2}x)$ is a stretch $\times 2$ horizontally



$f(2x)$ is a stretch $\times \frac{1}{2}$ horizontally



$-$ f(x) is a reflection in x-axis



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Sketching curves
Exercise F, Question 4

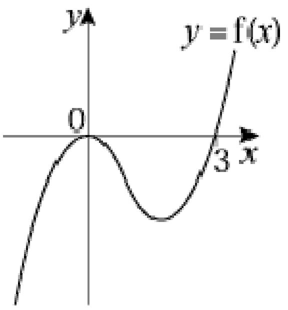
Question:

(a) Sketch the curve with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = x^2(x - 3)$.

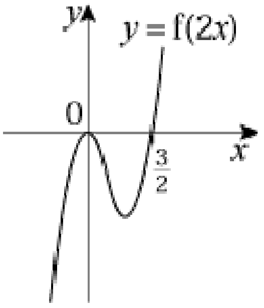
(b) Sketch the curves with equations $y = \mathrm{f}(2x)$, $y = - \mathrm{f}(x)$ and $y = \mathrm{f}(- x)$.

Solution:

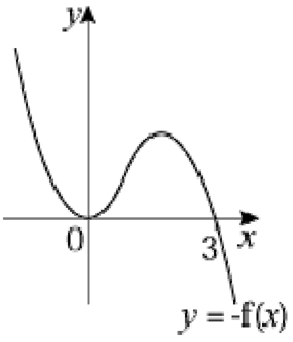
(a) $y = x^2(x - 3)$
$y = 0 \quad \Rightarrow \quad x = 0$ (twice), 3
Turning point at (0 , 0)
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



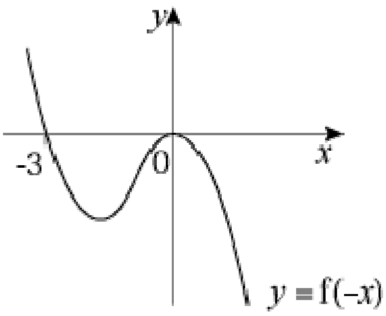
(b) f(2x) is a stretch $\times \frac{1}{2}$ horizontally



$-\mathrm{f}(x)$ is a reflection in x-axis



$f(-x)$ is a reflection in y-axis



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Sketching curves
Exercise F, Question 5

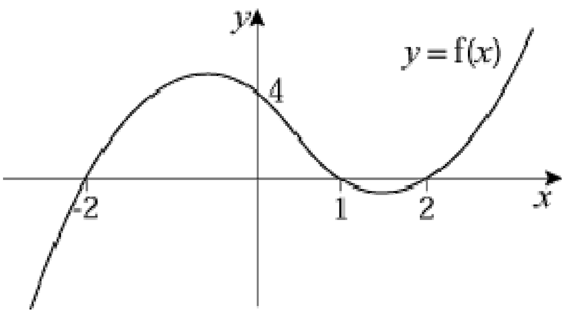
Question:

(a) Sketch the curve with equation $y = \text{f}(x)$ where $\text{f}(x) = (x - 2)(x - 1)(x + 2)$.

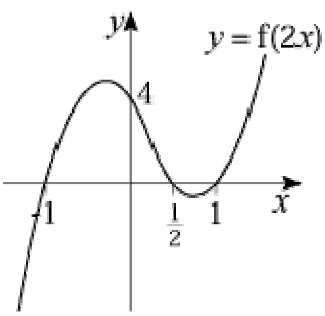
(b) Sketch the curves with equations $y = \text{f}(2x)$ and $\text{f}(\frac{1}{2}x)$.

Solution:

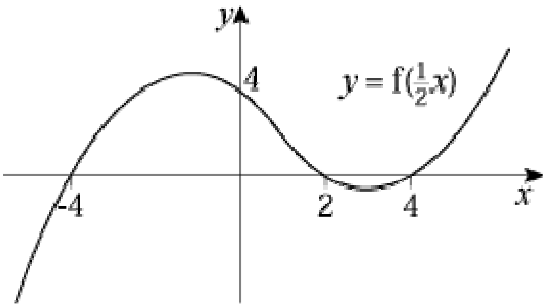
(a) $y = (x - 2)(x - 1)(x + 2)$
$y = 0 \Rightarrow x = 2, 1, -2$
$x = 0 \Rightarrow y = 4$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



(b) $\text{f}(2x)$ is a stretch $\times \frac{1}{2}$ horizontally



$\text{f}(\frac{1}{2}x)$ is a stretch $\times 2$ horizontally


y
4
$y = f(\frac{1}{2}x)$
-4
2
4
x

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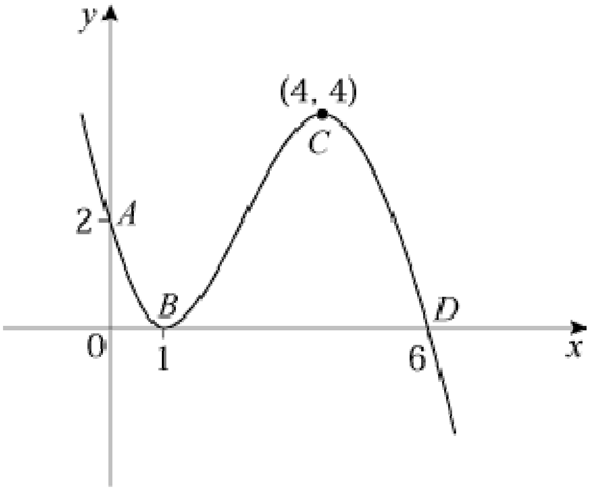
Sketching curves
Exercise G, Question 1

Question:

The following diagram shows a sketch of the curve with equation $y = f(x)$. The points $A(0, 2)$, $B(1, 0)$, $C(4, 4)$ and $D(6, 0)$ lie on the curve.



Sketch the following graphs and give the coordinates of the points A, B, C and D after each transformation:

(a) $f(x + 1)$

(b) $f(x) - 4$

(c) $f(x + 4)$

(d) $f(2x)$

(e) $3f(x)$

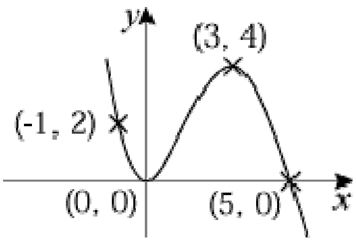
(f) $f(\frac{1}{2}x)$

(g) $\frac{1}{2}f(x)$

(h) $f(- x)$

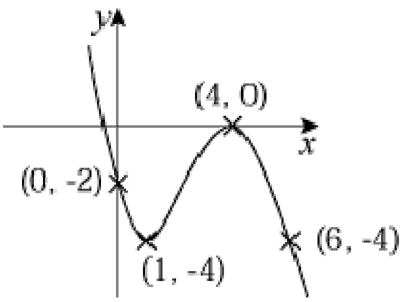
Solution:

(a) $f(x + 1)$ is a translation of -1 horizontally.



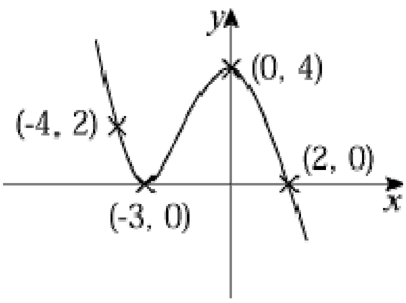
$A'(-1, 2)$; $B'(0, 0)$; $C'(3, 4)$; $D'(5, 0)$

(b) f(x) − 4 is a vertical translation of − 4.



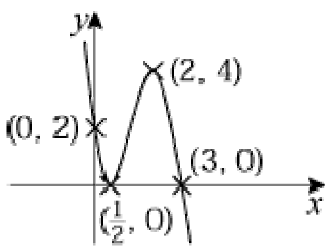
$A'(0, -2)$; $B'(1, -4)$; $C'(4, 0)$; $D'(6, -4)$

(c) f(x + 4) is a translation of − 4 horizontally.



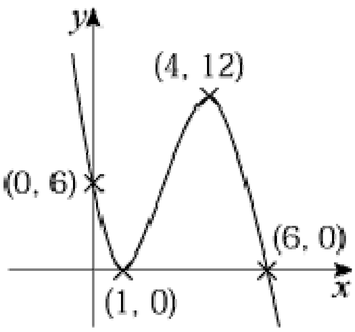
$A'(-4, 2)$; $B'(-3, 0)$; $C'(0, 4)$; $D'(2, 0)$

(d) f(2x) is a stretch of $\frac{1}{2}$ horizontally.



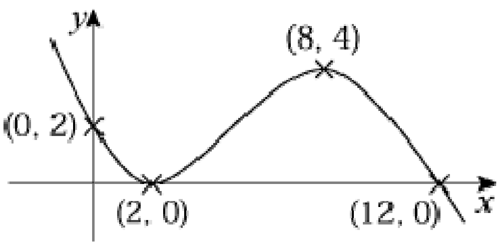
$A'(0, 2)$; $B'(\frac{1}{2}, 0)$; $C'(2, 4)$; $D'(3, 0)$

(e) 3f(x) is a stretch of 3 vertically.



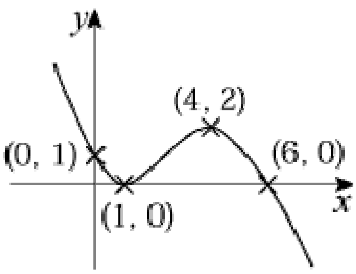
A′(0 , 6); *B′*(1 , 0); *C′*(4 , 12); *D′*(6 , 0)

(f) f($\frac{1}{2}x$) is a stretch of 2 horizontally.



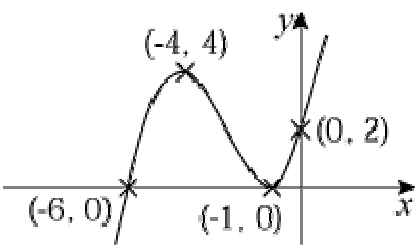
A′(0 , 2); *B′*(2 , 0); *C′*(8 , 4); *D′*(12 , 0)

(g) $\frac{1}{2}$f(*x*) is a stretch of $\frac{1}{2}$ vertically.



A′(0 , 1); *B′*(1 , 0); *C′*(4 , 2); *D′*(6 , 0)

(h) f(− *x*) is a reflection is the *y*-axis.



A′(0 , 2); *B′*(− 1 , 0); *C′*(− 4 , 4); *D′*(− 6 , 0)

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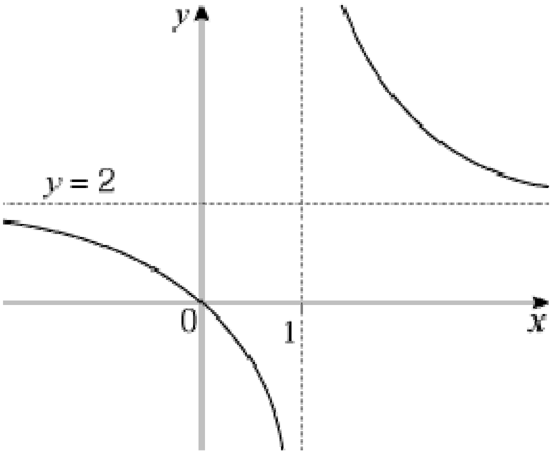
Sketching curves
Exercise G, Question 2

Question:

The curve $y = f(x)$ passes through the origin and has horizontal asymptote $y = 2$ and vertical asymptote $x = 1$, as shown in the diagram.



Sketch the following graphs and give the equations of any asymptotes and, for all graphs except (a), give coordinates of intersections with the axes after each transformation.

(a) $f(x) + 2$

(b) $f(x + 1)$

(c) $2f(x)$

(d) $f(x) - 2$

(e) $f(2x)$

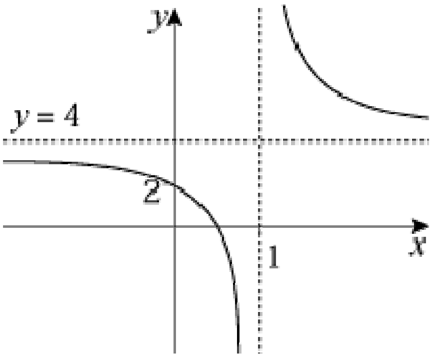
(f) $f(\frac{1}{2}x)$

(g) $\frac{1}{2}f(x)$

(h) $-f(x)$

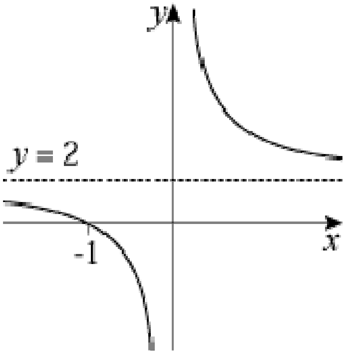
Solution:

(a) $f(x) + 2$ is a translation of $+ 2$ in a vertical direction.



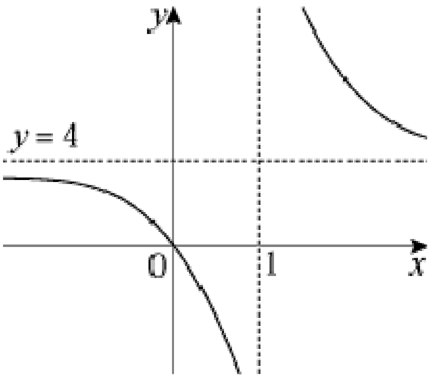
Asymptotes: $x = 1$, $y = 4$. Intersections: $(0, 2)$ and $(a, 0)$, where $0 < a < 1$.

(b) $f(x + 1)$ is a horizontal translation of -1.



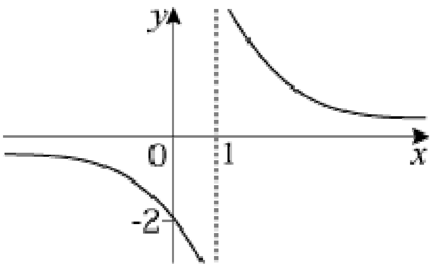
Asymptotes: $x = 0$, $y = 2$. Intersections: $(-1, 0)$

(c) $2f(x)$ is a stretch of 2 in a vertical direction.



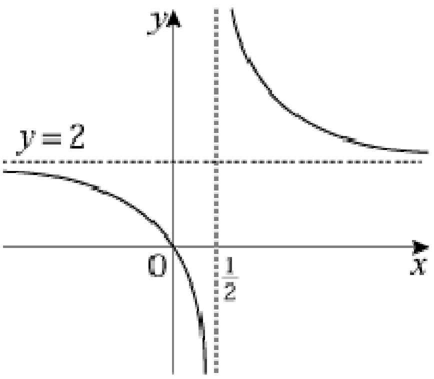
Asymptotes: $x = 1$, $y = 4$. Intersections: $(0, 0)$

(d) $f(x) - 2$ is a vertical translation of -2.



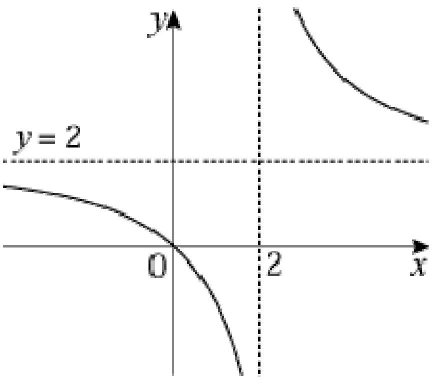
Asymptotes: $x = 1, y = 0$. Intersections: $(0\ ,\ -2)$

(e) f(2x) is a stretch of $\frac{1}{2}$ in a horizontal direction.



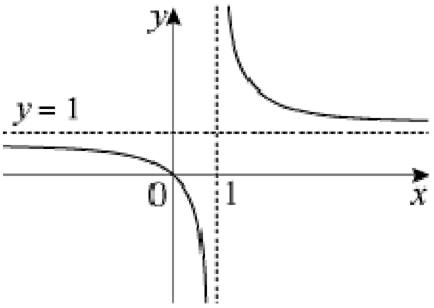
Asymptotes: $x = \frac{1}{2}, y = 2$. Intersections: $(0\ ,\ 0)$

(f) f($\frac{1}{2}x$) is a stretch of 2 in a horizontal direction.



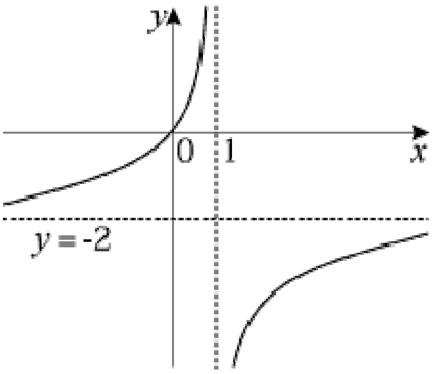
Asymptotes: $x = 2, y = 2$. Intersections: $(0\ ,\ 0)$

(g) $\frac{1}{2}$f(x) is a stretch of $\frac{1}{2}$ in a vertical direction.



Asymptotes: $x = 1$, $y = 1$. Intesections: (0 , 0)

(h) $-\mathrm{f}(x)$ is a reflection in the x-axis.



Asymptotes: $x = 1$, $y = -2$. Intersections: (0 , 0)

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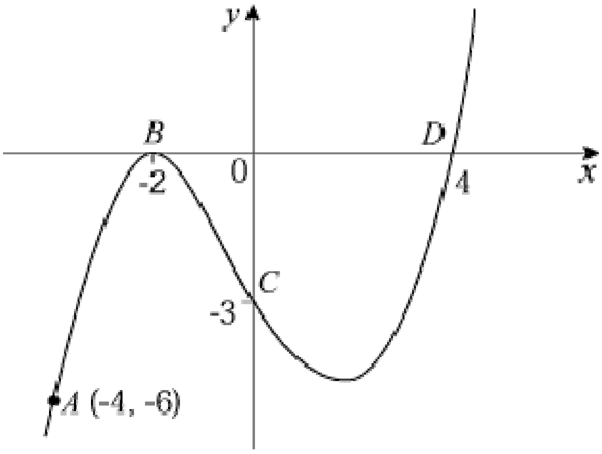
Edexcel Modular Mathematics for AS and A-Level

Sketching curves
Exercise G, Question 3

Question:

The curve with equation $y = f(x)$ passes through the points $A(-4, -6)$, $B(-2, 0)$, $C(0, -3)$ and $D(4, 0)$ as shown in the diagram.



Sketch the following and give the coordinates of the points *A*, *B*, *C* and *D* after each transformation.

(a) $f(x-2)$

(b) $f(x)+6$

(c) $f(2x)$

(d) $f(x+4)$

(e) $f(x)+3$

(f) $3f(x)$

(g) $\frac{1}{3}f(x)$

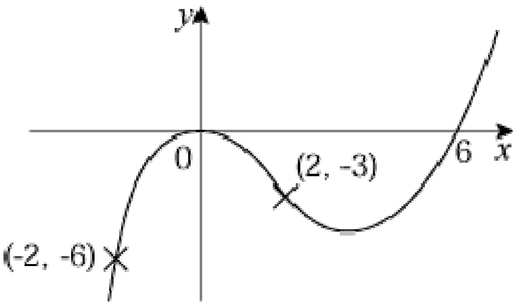
(h) $f(\frac{1}{4}x)$

(i) $-f(x)$

(j) $f(-x)$

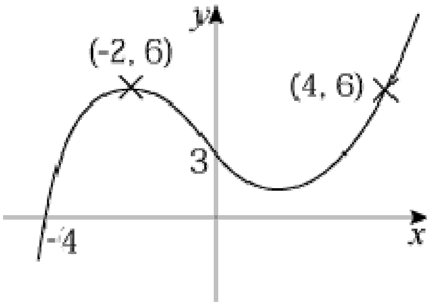
Solution:

(a) $f(x-2)$ is a horizontal translation of $+2$.



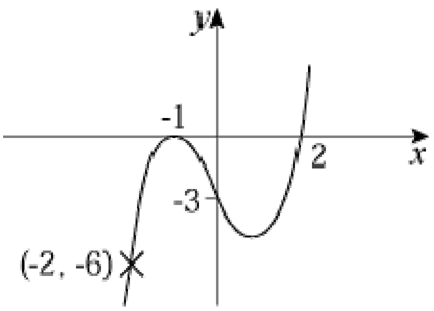
A′(− 2 , − 6); *B*′(0 , 0); *C*′(2 , − 3); *D*′(6 , 0)

(b) f(*x*) + 6 is a vertical translation of + 6.



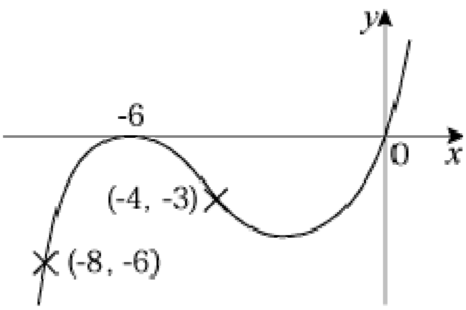
A′(− 4 , 0); *B*′(− 2 , 6); *C*′(0 , 3); *D*′(4 , 6)

(c) f(2*x*) is a horizontal stretch of $\frac{1}{2}$.



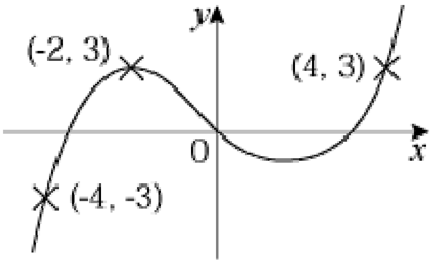
A′(− 2 , − 6); *B*′(− 1 , 0); *C*′(0 , − 3); *D*′(2 , 0)

(d) f(*x* + 4) is a horizontal translation of − 4.



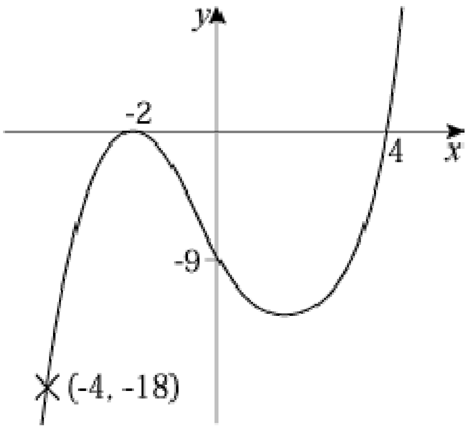
A′(− 8 , − 6); *B*′(− 6 , 0); *C*′(− 4 , − 3); *D*′(0 , 0)

(e) f(*x*) + 3 is a vertical translation of + 3.



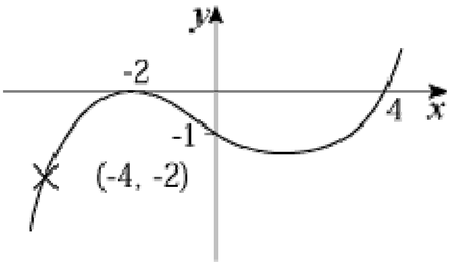
$A'(-4, -3)$; $B'(-2, 3)$; $C'(0, 0)$; $D'(4, 3)$

(f) $3f(x)$ is a vertical stretch of 3.



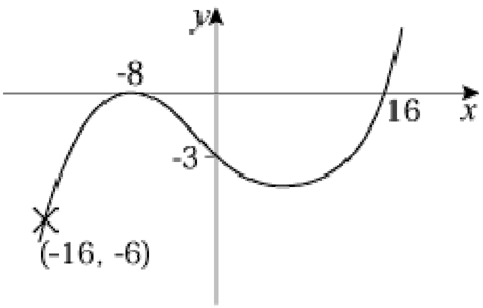
$A'(-4, -18)$; $B'(-2, 0)$; $C'(0, -9)$; $D'(4, 0)$

(g) $\frac{1}{3}f(x)$ is a vertical stretch of $\frac{1}{3}$.



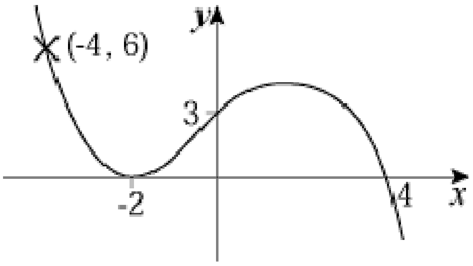
$A'(-4, -2)$; $B'(-2, 0)$; $C'(0, -1)$; $D'(4, 0)$

(h) $f(\frac{1}{4}x)$ is a horizontal stretch of 4.



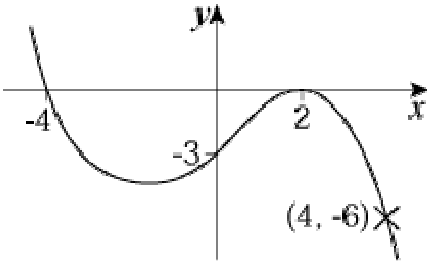
$A'(-16, -6)$; $B'(-8, 0)$; $C'(0, -3)$; $D'(16, 0)$

(i) $-f(x)$ is a reflection in the x-axis.



$A'(-4, 6)$; $B'(-2, 0)$; $C'(0, 3)$; $D'(4, 0)$

(j) $f(-x)$ is a reflection in the y-axis.



$A'(4, -6)$; $B'(2, 0)$; $C'(0, -3)$; $D'(-4, 0)$

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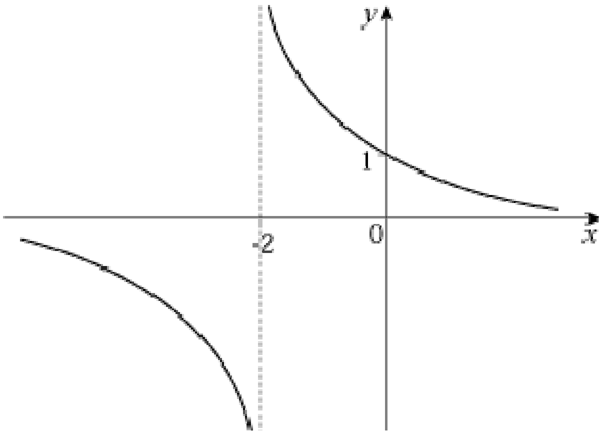
Sketching curves
Exercise G, Question 4

Question:

A sketch of the curve $y = f(x)$ is shown in the diagram. The curve has vertical asymptote $x = -2$ and a horizontal asymptote with equation $y = 0$. The curve crosses the y-axis at $(0, 1)$.



(a) Sketch, on separate diagrams, the graphs of:

(i) $2f(x)$

(ii) $f(2x)$

(iii) $f(x - 2)$

(iv) $f(x) - 1$

(v) $f(-x)$

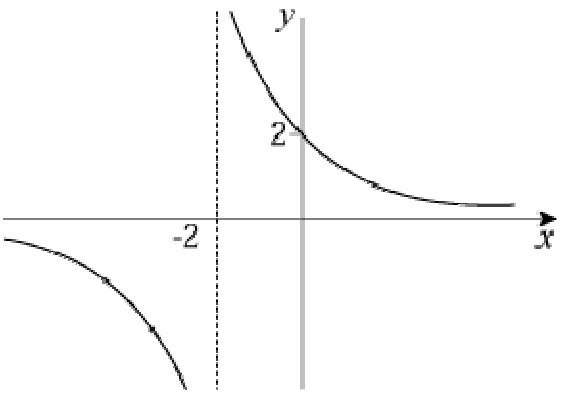
(vi) $-f(x)$

In each case state the equations of any asymptotes and, if possible, points where the curve cuts the axes.

(b) Suggest a possible equation for $f(x)$.

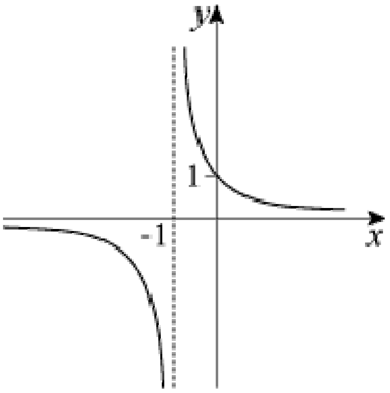
Solution:

(a) (i) $2f(x)$ is a vertical stretch of 2.



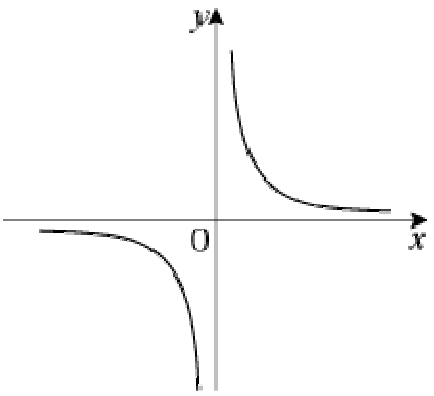
Asymptotes: $x = -2$, $y = 0$. Intersections: $(0, 2)$

(ii) f(2*x*) is a horizontal stretch of $\frac{1}{2}$.



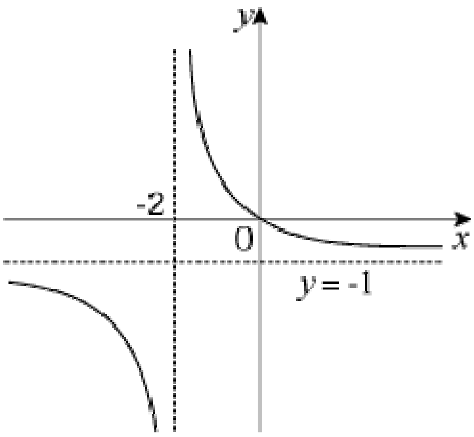
Asymptotes: $x = -1$, $y = 0$. Intersections: $(0, 1)$

(iii) $f(x - 2)$ is a translation of $+2$ in the *x*-direction.



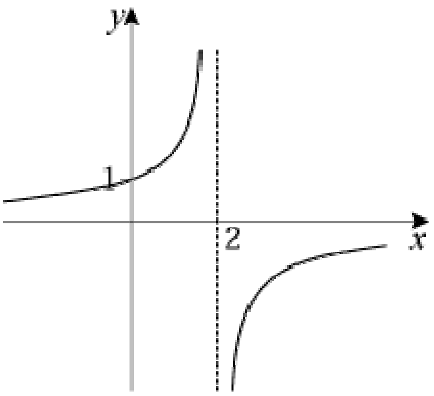
Asymptotes: $x = 0$, $y = 0$. No intersections with axes.

(iv) $f(x) - 1$ is a translation of -1 in the *y*-direction.



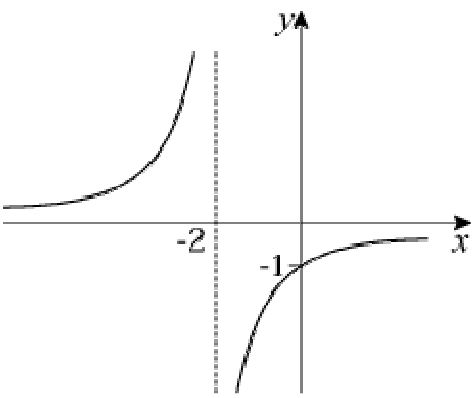
Asymptotes: $x = -2$, $y = -1$. Intersections: (0 , 0)

(v) $f(-x)$ is a reflection in the y-axis.



Asymptotes: $x = 2$, $y = 0$. Intersections: (0 , 1)

(vi) $-f(x)$ is a reflection in the x-axis.



Asymptotes: $x = -2$, $y = 0$. Intersections: $(0, -1)$

(b) The shape of the curve is like $y = \frac{k}{x}$, $k > 0$.

$x = -2$ asymptote suggests denominator is zero when $x = -2$, so denominator is $x + 2$.
Also, $f(0) = 1$ means 2 required on numerator.

$f(x) = \frac{2}{x + 2}$

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Sketching curves
Exercise H, Question 1

Question:

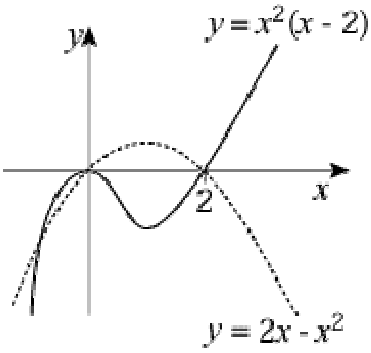
(a) On the same axes sketch the graphs of $y = x^2(x - 2)$ and $y = 2x - x^2$.

(b) By solving a suitable equation find the points of intersection of the two graphs.

Solution:

(a) $y = x^2(x - 2)$
$y = 0 \Rightarrow x = 0$ (twice), 2
Turning point at (0 , 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$

$y = 2x - x^2 = x(2 - x)$ is $\cap$ shaped
$y = 0 \Rightarrow x = 0, 2$



(b) $x^2(x - 2) = x(2 - x)$
$\Rightarrow x^2(x - 2) - x(2 - x) = 0$
$\Rightarrow x(x - 2)(x + 1) = 0$
$\Rightarrow x = 0, 2, -1$
Using $y = x(2 - x)$ the points of intersection are:
(0 , 0); (2 , 0); (− 1 , − 3)

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Sketching curves
Exercise H, Question 2

Question:

(a) On the same axes sketch the curves with equations $y = \frac{6}{x}$ and $y = 1 + x$.

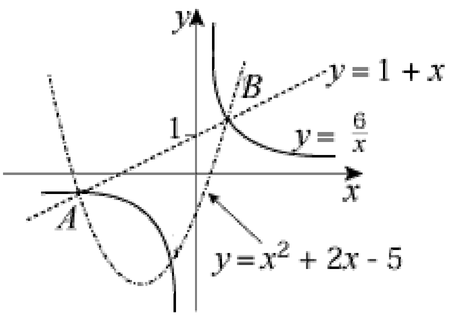
(b) The curves intersect at the points A and B. Find the coordinates of A and B.

(c) The curve C with equation $y = x^2 + px + q$, where p and q are integers, passes through A and B. Find the values of p and q.

(d) Add C to your sketch.

Solution:

(a) $y = \frac{6}{x}$ is like $y = \frac{1}{x}$ and $y = 1 + x$ is a straight line.



(b) $\frac{6}{x} = 1 + x$

$\Rightarrow \quad 6 = x + x^2$

$\Rightarrow \quad 0 = x^2 + x - 6$

$\Rightarrow \quad 0 = (x + 3)(x - 2)$

$\Rightarrow \quad x = 2, \; -3$

So A is $(-3, -2)$; B is $(2, 3)$

(c) Substitute the points A and B into $y = x^2 + px + q$:

$A \quad \Rightarrow \quad -2 = 9 - 3p + q$ ①

$B \quad \Rightarrow \quad 3 = 4 + 2p + q$ ②

① − ②: $-5 = 5 - 5p$

$\Rightarrow \quad p = 2$

$\Rightarrow \quad q = -5$

(d) $y = x^2 + 2x - 5 = (x + 1)^2 - 6 \quad \Rightarrow \quad$ minimum at $(-1, -6)$

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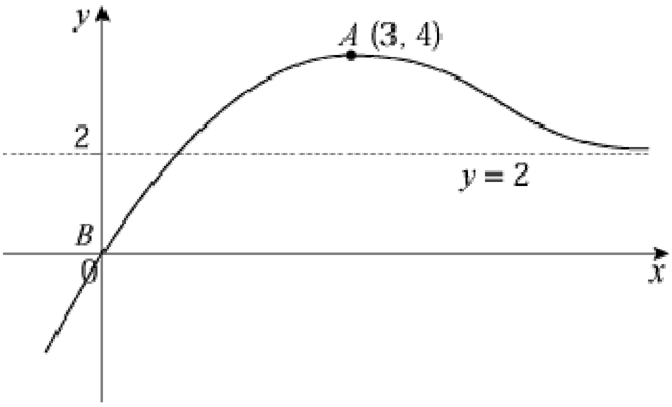
Sketching curves
Exercise H, Question 3

Question:

The diagram shows a sketch of the curve $y = f(x)$. The point $B(0, 0)$ lies on the curve and the point $A(3, 4)$ is a maximum point. The line $y = 2$ is an asymptote.



Sketch the following and in each case give the coordinates of the new positions of A and B and state the equation of the asymptote:

(a) $f(2x)$

(b) $\frac{1}{2}f(x)$

(c) $f(x) - 2$

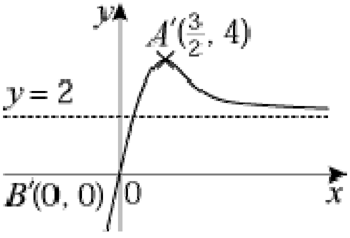
(d) $f(x + 3)$

(e) $f(x - 3)$

(f) $f(x) + 1$

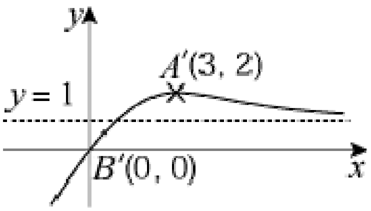
Solution:

(a) $f(2x)$ is a horizontal stretch of $\frac{1}{2}$.



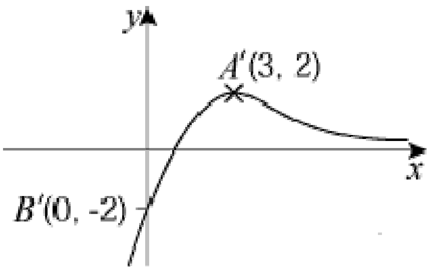
$A'(\frac{3}{2}, 4)$; $B'(0, 0)$. Asymptote: $y = 2$.

(b) $\frac{1}{2}$f(x) is a vertical stretch of $\frac{1}{2}$.



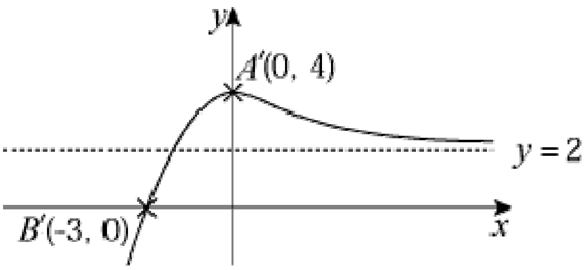
A'(3 , 2); B'(0 , 0). Asymptote: y = 1.

(c) f(x) − 2 is a vertical translation of − 2.



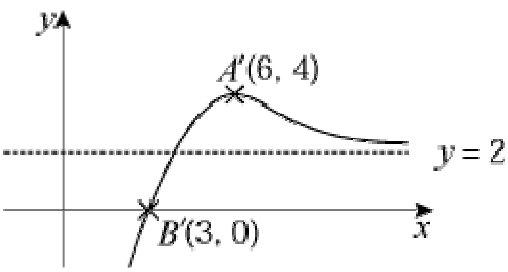
A'(3 , 2); B'(0 , − 2). Asymptote: y = 0.

(d) f(x + 3) is a horizontal translation of − 3.



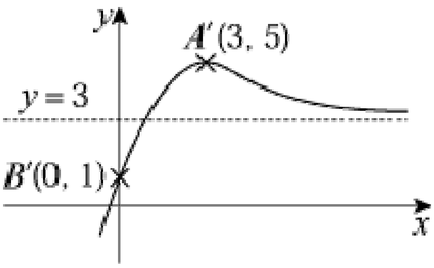
A'(0 , 4); B'(− 3 , 0). Asymptote: y = 2.

(e) f(x − 3) is a horizontal translation of + 3.



A'(6 , 4); B'(3 , 0). Asymptote: y = 2.

(f) f(x) + 1 is a vertical translation of + 1.



A′(3 , 5); *B′*(0 , 1). Asymptote: $y = 3$.

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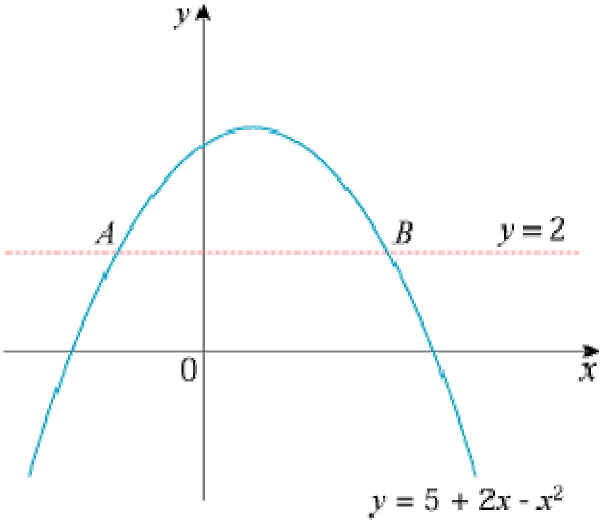
Sketching curves
Exercise H, Question 4

Question:

The diagram shows the curve with equation $y = 5 + 2x - x^2$ and the line with equation $y = 2$. The curve and the line intersect at the points A and B.



Find the x-coordinates of A and B. **[E]**

Solution:

$2 = 5 + 2x - x^2$
$x^2 - 2x - 3 = 0$
$(x - 3)(x + 1) = 0$
$x = -1, 3$

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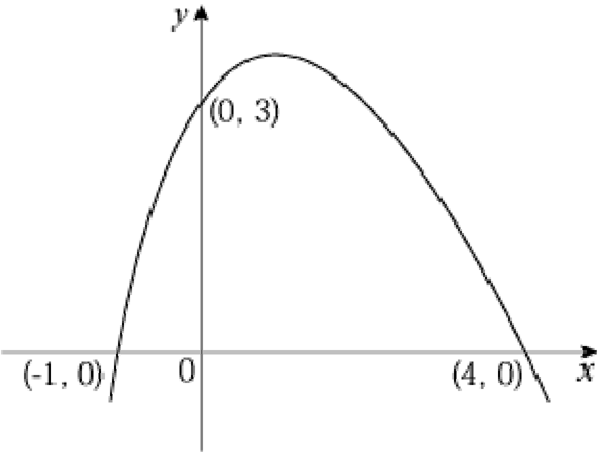
Sketching curves
Exercise H, Question 5

Question:

The curve with equation $y = f(x)$ meets the coordinate axes at the points $(-1, 0)$, $(4, 0)$ and $(0, 3)$, as shown in the diagram.



Using a separate diagram for each, sketch the curve with equation

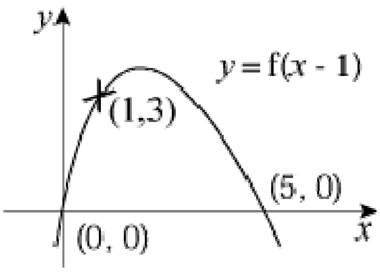
(a) $y = f(x - 1)$

(b) $y = -f(x)$

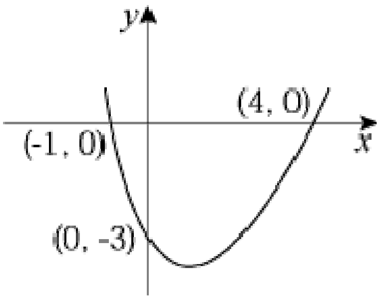
On each sketch, write in the coordinates of the points at which the curve meets the coordinate axes. **[E]**

Solution:

(a) $f(x - 1)$ is a translation of $+1$ in the x-direction.



(b) $-f(x)$ is a reflection in the x-axis.


y
x
(4, 0)
(-1, 0)
(0, -3)

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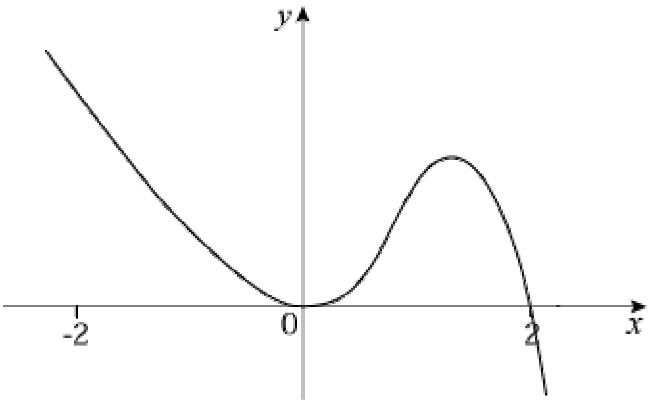
Sketching curves
Exercise H, Question 6

Question:

The figure shows a sketch of the curve with equation $y = f(x)$.



In separate diagrams show, for $-2 \leq x \leq 2$, sketches of the curves with equation:

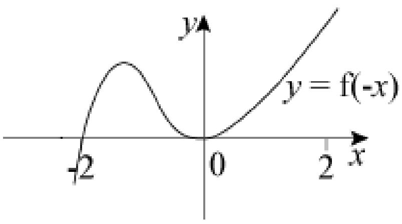
(a) $y = f(-x)$

(b) $y = -f(x)$

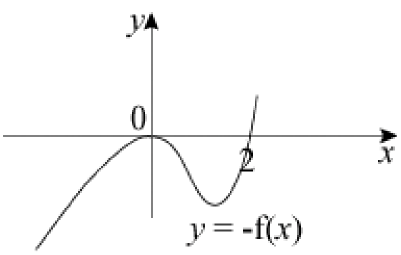
Mark on each sketch the x-coordinate of any point, or points, where a curve touches or crosses the x-axis. **[E]**

Solution:

(a) $f(-x)$ is a reflection in the y-axis.



(b) $-f(x)$ is a reflection in the x-axis.



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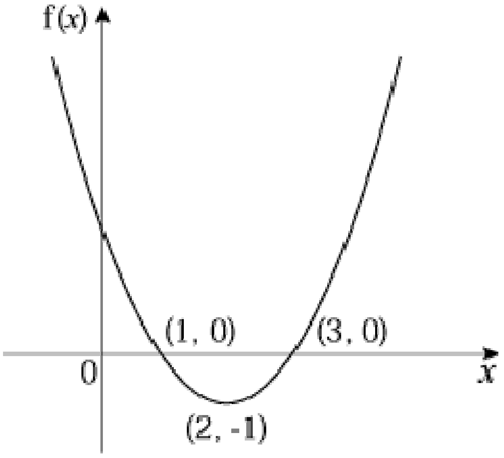
Sketching curves
Exercise H, Question 7

Question:

The diagram shows the graph of the quadratic function f. The graph meets the x-axis at (1 , 0) and (3 , 0) and the minimum point is (2 , − 1).



(a) Find the equation of the graph in the form $y = f(x)$.

(b) On separate axes, sketch the graphs of

(i) $y = f(x + 2)$

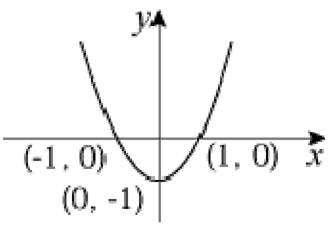
(ii) $y = f(2x)$

(c) On each graph write in the coordinates of the points at which the graph meets the x-axis and write in the coordinates of the minimum point. **[E]**

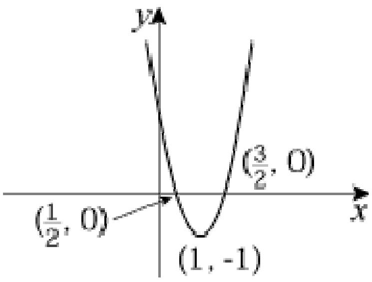
Solution:

(a) Let $y = a(x - p)(x - q)$
Since (1 , 0) and (3 , 0) are on the curve then $p = 1$, $q = 3$
So $y = a(x - 1)(x - 3)$
Using (2 , − 1) $\Rightarrow$ $-1 = a(1)(-1)$ $\Rightarrow$ $a = 1$
So $y = (x - 1)(x - 3) = x^2 - 4x + 3$

(b) (i) $f(x + 2) = (x + 1)(x - 1)$, or translation of − 2 in the x-direction.



(ii) $f(2x) = (2x - 1)(2x - 3)$, or horizontal stretch of $\frac{1}{2}$.


y
x
$(\frac{3}{2}, 0)$
$(\frac{1}{2}, 0)$
(1, -1)